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PROCEEDINGS

THIRD MEETING, SECOND HALF, 66TH SESSION

A joint meeting of the Royal Aeronautical Society and the British Gliding Association was held in the Lecture Hall of the Royal Society of Arts, 18, John Street, Adelphi, London, W.C.2, on Thursday, January 29th, 1931, when a paper on "The Development, Design and Construction of Gliders and Sailplanes," by Herr A. Lippisch, was read and discussed. The President of the Royal Aeronautical Society (Mr. C. R. Fairey) was in the chair, and was accompanied on the platform by the Chairman of the British Gliding Association (Mr. C. Gordon England).

The CHAIRMAN: On behalf of the Royal Aeronautical Society, he took the opportunity very heartily to congratulate the British Gliding Association upon having done so much, under the energetic chairmanship of Mr. Gordon England, to advance the art of gliding and soaring in this country in so short a space of time. The art of gliding and soaring, he continued, was very much in the minds of all interested in aviation. It had provided a new sport, it had provided scope for the designer, and was making, and no doubt would continue to make, as the result of research, many useful contributions to the development of aeronautics.

There was no subject more suitable for discussion at such a meeting than that of the development, design and construction of gliders and sailplanes, and there was no finer exponent of the subject in the whole world than Herr Lippisch, Director of the Technical Department of the Rhön-Rossitten Gesellschaft, the headquarters of the ruling gliding authority in Germany—and, from the days of Lilienthal, it was in Germany that the greatest development of this art had been made. Herr Lippisch had taken one of the greatest parts in that development, perhaps a greater part than any other man. He had served in the Great War, and since the war had devoted himself entirely to the technical practice of aviation, chiefly in relation to the art of soaring. The Chairman added that he had learned, with surprise and gratitude, that Herr Lippisch was the designer of, among other things, some of the famous Göttingen wing sections, which many British designers were using. He was also the designer of many successful sailplanes, too numerous to mention. In the paper which Herr Lippisch had prepared for presentation to the meeting he had included, without reserve, most valuable information collected over a number of years.

THE DEVELOPMENT, DESIGN AND CONSTRUCTION OF GLIDERS AND SAILPLANES

BY

A. LIPPISCH

Introduction

The Gliding and Soaring Flight movement is as old as History. The observation of bird flight must have shown the men of former centuries in the same way as our own generation, that apart from power flight with wing beats there must be another flight possibility, enabling the use of the energy in the movement of air masses for flight without the expenditure of other power. The experiments undertaken in these times on the solution of the flight problem have come down to us only through myths and sagas, and we cannot differentiate between truth and imagination in these stories.

Experimental research, which can certainly be considered as the foundation of modern physics, has also in the realm of aerodynamics laid the basis for modern

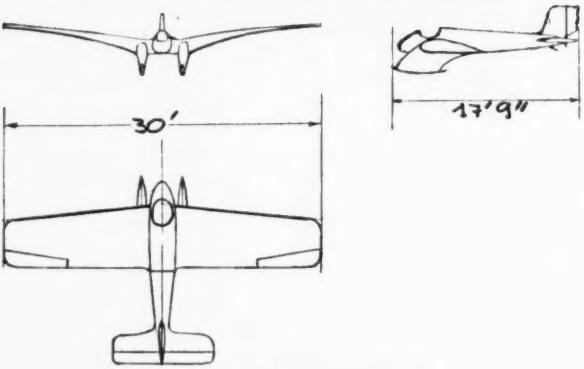


FIG. 1.

Schwarzer Teufel.

Empty weight, 133lb. Wing area, 161 sq. ft.

aeronautics. In this gliding and soaring flight plays the rôle of full-scale experiments, not as an end in itself, but as a proving-ground and last station before the invention of power flight.

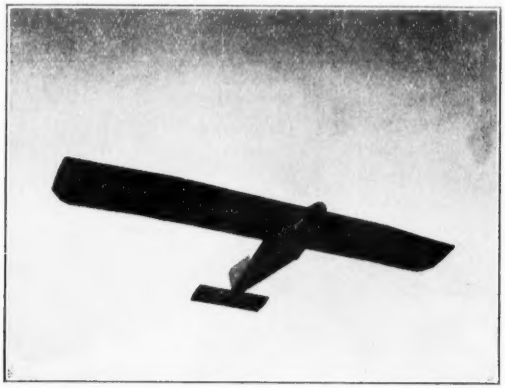
The actual gliding and sailing flying had its beginning in the sailing flight movement which took place after the war and which was a result of the Rhoen Sailplane Contests.

Part I.

So if I commence my lecture with a few remarks about the development of the gliding and sailing flight movement, I will begin this outline with the successful sailplane of the first Rhoen Sailplane Contest.

Fig. 1 shows the aircraft " Schwarzer Teufel " (Black Devil) of the Aachen Flying Club, designed by W. Klemperer.

The underlying idea of the design is on one side low structural weight and on the other side the greatest possible reduction of the parasite drag. Especially



The Vampyr.

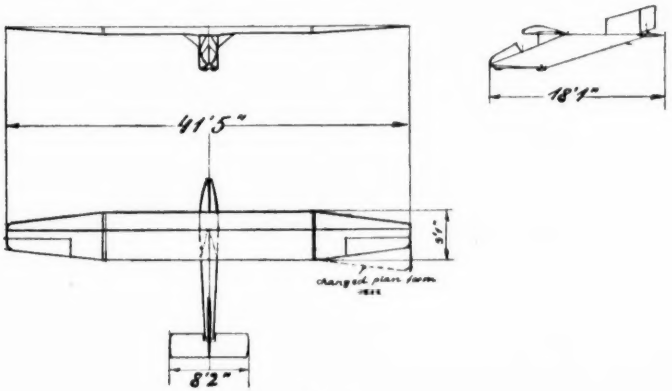


FIG. 2.

Vampyr.

Empty weight, 287lbs. Wing area, 172 sq. ft.

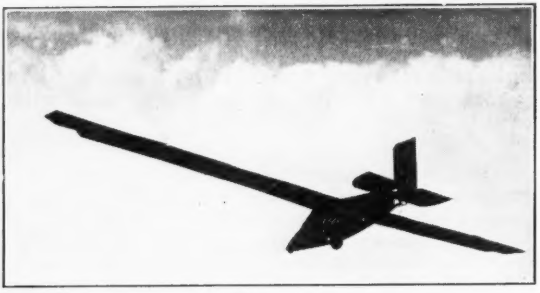


FIG. 3.
Espenlaub's machine.

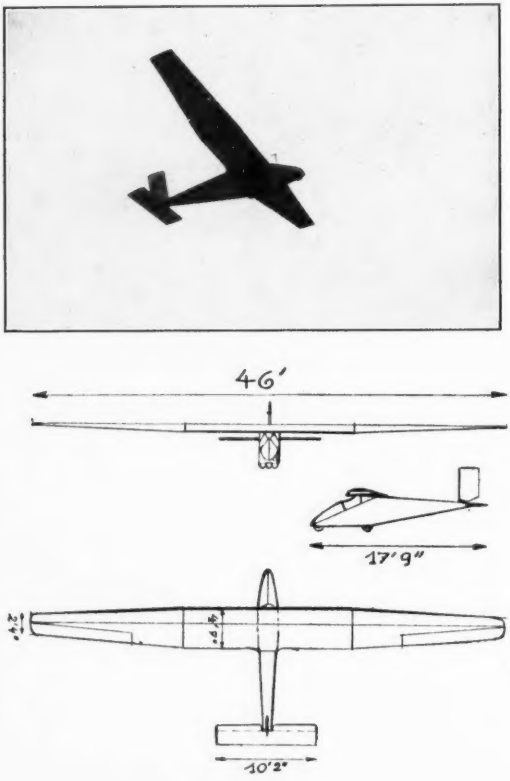


FIG. 3A.
Strolch (1923).
Empty weight, 183lbs. *Wing area*, 151 *sq. ft.*

notable in this and other designs by Klemperer is the unusually carefully carried out structure. By this means Klemperer was able to attain an empty weight for this machine of 133 lb. (61 kg.) or a wing loading of 1.86 lb./sq. ft.

This type was not copied in the years following due to the fact that although the sinking speed was satisfactory, the gliding angle was too great. This matter depends to a considerable extent on the low-wing construction.

The sailplane which has to-day almost become a classic is the "Vampyr" (Fig. 2) of the Flying Club of the Hannover Engineering School, which was designed by G. Madelung.

In this design we find for the first time the essential lines of thought clearly worked out in the design of a sailplane.

The Konsul.

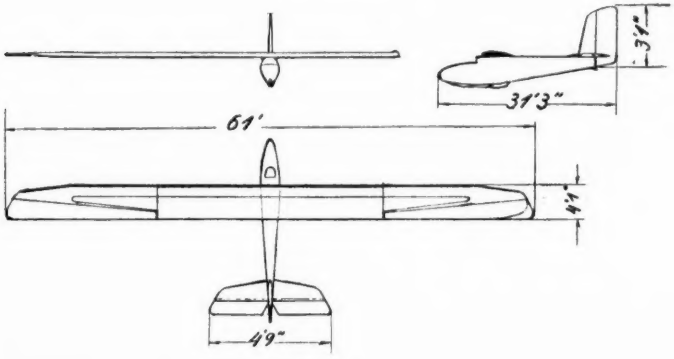


FIG. 4.

Konsul, Darmstadt.

Empty weight, 418lb. Wing area, 232 sq. ft.

The problem is to build an aircraft with low sinking speed, good gliding angle, sufficient strength and good manœuvrability.

The solution is: A cantilever high-wing type with a thick highly cambered wing section with large span and aspect ratio.

In this type a single-spar wing was used for the first time in which the torsional forces were taken by the leading edge. This was built up as a thin-walled tube, closed at the rear by the spar proper. This " torsion-nose " allowed at the same time of the possibility of keeping the most sensitive part of the wing section, the leading edge, the exactly correct shape. Plywood is the essential material which first made this construction possible.

Further notable characteristics of the " Vampyr " are: The three-part wing, the fuselage completely built of plywood and with an undercarriage consisting of three football wheels, and the pendulum-type elevator.

The complete superiority of the " Vampyr " in the contests of 1921-22 shows the correctness of the way that was taken.

It is obvious that the following years would bring a further development of the " Vampyr " idea.

Roemryke Berge.
FIG. 5.

At this time a rivalry sprang up to see who could get the greatest span and the best aspect ratio.

The first who dared to build with large span, cantilever, and with relatively thin wings, disregarding the necessarily appearing deflection due to bending, was the cabinet-maker Espenlaub (Fig. 3).

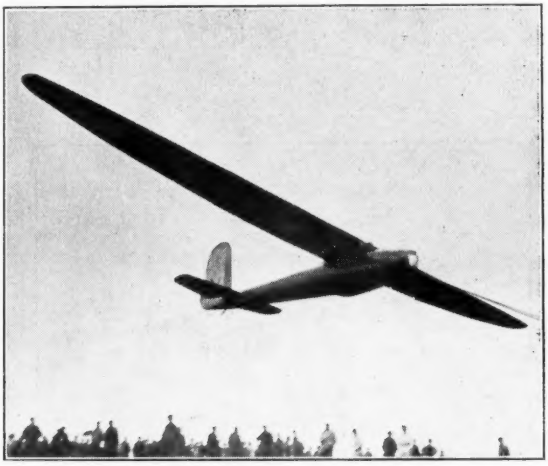
Even although the aircraft concerned was not successful, it set the example for the successful designs " Strolch " and " Konsul."

The " Strolch " (Fig. 3a), which was designed by A. Martens, is a refined development of the " Vampyr " with lower empty weight, increased span, and further, lessened parasite drag. Special care was taken over the rapid and simple assembly, an important point when regarding the use in contests.

As competitor the " Konsul " (Fig. 4) of the Darmstadt Academic Flying Club also appeared in 1923. This aircraft, which was designed by Botch and Spiess, began the series of sailplanes which have come from the Darmstadt School. Certain differences from the " Vampyr " are unmistakable. The span is considerably increased, the wing-plan approaches the elliptic form, the fuselage is round in section, the empennage is considerably enlarged, and divided into fin and rudder. So as to guarantee the aileron effect with the great span the rudder was differentially connected with the ailerons. The " Konsul " may

rightly be considered to be the first long-distance sailplane. The flying qualities of this aircraft were considered excellent by all pilots. The aircraft which surpassed the performances of the " Konsul " was also a design of a Darmstadt student. This was the sailplane " Roemryke Berge " (Fig. 5) which was designed by Schatzky in 1924 for the Niederrheinischer Verein fuer Luftfahrt.

This machine hardly appeared in 1924 and 1925, as the Club had no suitable pilot for it. J. Nehring was the first to master this aircraft which was undoubtedly difficult to handle, and by his flight to the Milseburg proved the superiority of this machine. The beautifully streamlined fuselage had the smallest possible cross-section. The wing rested on the narrow base of a centre-piece of the fuselage which served at the same time as streamlining behind the pilot's head. The wing section was Goettingen 426, a slightly cambered, thin section, with remarkably small drag, and the aspect ratio was suitable for the wing section.



The Darmstadt II.

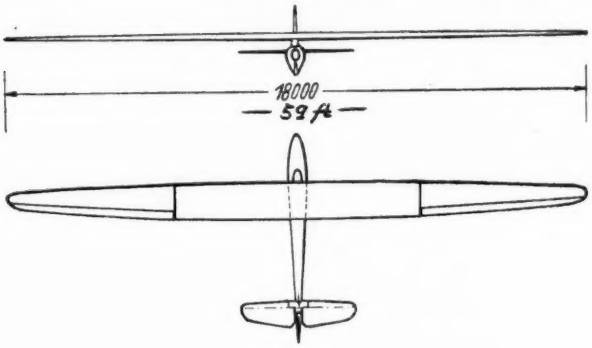
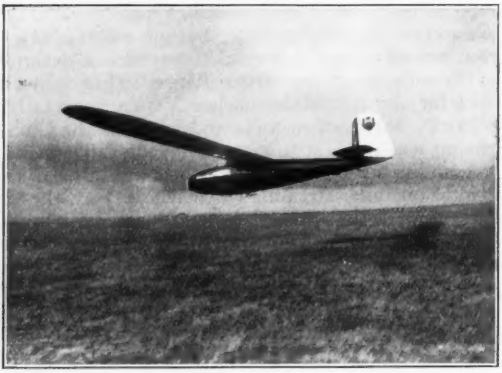


FIG. 6.

Darmstadt II.
A = 19.6.
S = 182 *sq. ft.*
W = 253*lb.*



Westpreussen.
FIG. 7.

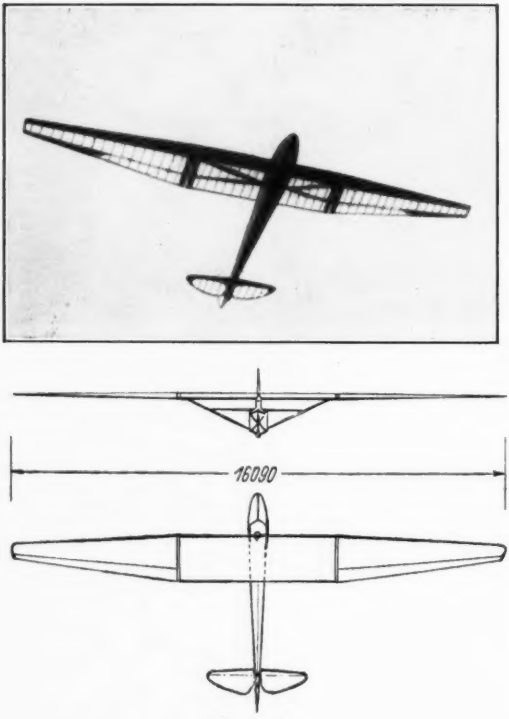
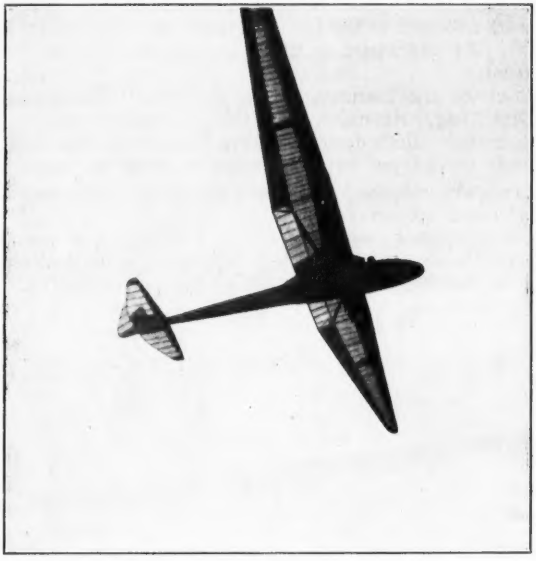


FIG. 8.
Rhöngeist (Professor).
$A = 13.9$.
$S = 203$ *sq. ft.*
$W = 334$ *lb.*

The design of a sailplane from this latter point of view will be more exactly investigated later.

The centre-section had a rectangular plan-form and a flap along the whole trailing edge, which could be adjusted to vary the wing section. This mechanism was, however, never used and was later removed. Next year (1927) the Darmstadt



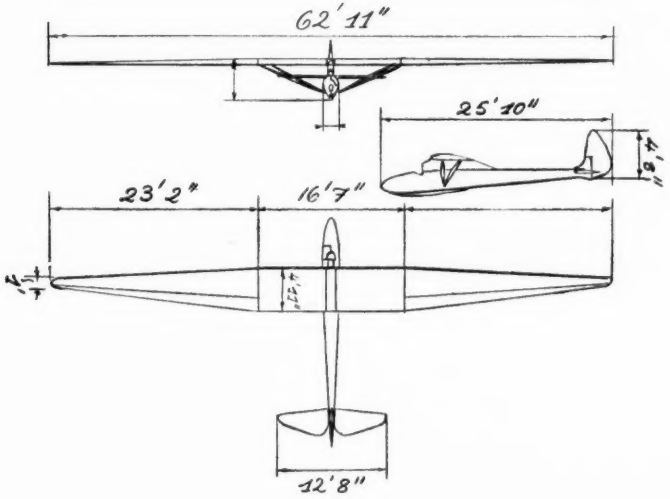


FIG. 8A.

Wien.

Empty weight, 353lb. Aspect ratio 20.

Club built the type " Darmstadt I" to take the place of the decrepit " Konsul," from which it was developed. The influence of the " Roemryke Berge " is as regards the position of the pilot, unmistakable. The work of development in this case and also in the types that followed is directed more on the lines of the technical details. The " Darmstadt I" is smaller than the " Konsul " and, therefore, for contest purposes more useful, and as a result of the smaller structural weight no less capable of good performances.

As the aircraft was sold to the United States, the club built " Darmstadt II " (Fig. 6) in 1928. As compared to the " Darmstadt I " the wing section and span were different.

As a variation of the Darmstadt type are the " Westpreussen " (Fig. 7) designed by Dipl. Ing. Hofmann and the " Wuerttemberg " and " Lore " designed by Laubenthal. Both designers were formerly members of the Darmstadt Club. Accordingly these types have a number of points in common, for example:

Wing.—Three-part, single-spar, rectangular (plan) centre section with tapered elliptical outer sections.

Fuselage.—Egg-shaped cross-section, with sharp edge underneath, cabane compact or non-existent, fuselage ending in a vertical edge.

Empennage.—Attached to small built-in fin and tailplane. Otherwise no damping surfaces.

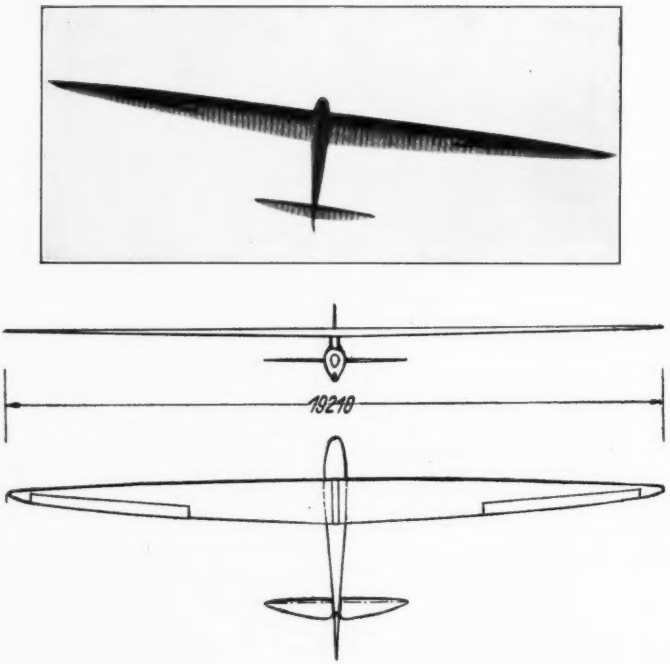


FIG. 8B.
Kakadu.
$A = 21$.
$S = 190$ sq. ft.
$W = 298lb$.

It is easily understood that this wing plan-form is uneconomical to build after reaching a certain span, as the spar depth would not be great enough for the cantilever type.

To avoid this difficulty from the beginning, the lecturer designed the type " Professor " (Fig. 8) in 1928 as a high-performance training aircraft. I consciously departed from the usual cantilever construction with compact short fuselage. As the aircraft was designed to be built by the various clubs and for training, the strength and stiffness had to be as great as possible. This is easiest

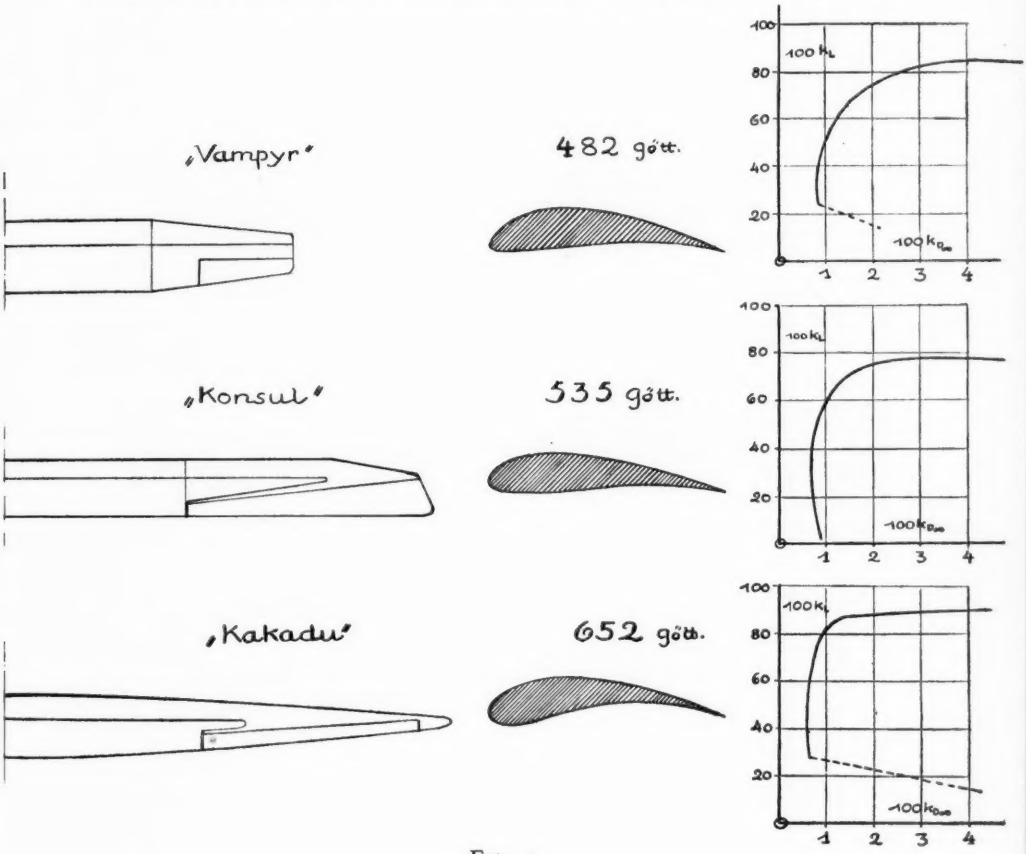


FIG. 9.

to achieve by using a braced wing and a long fuselage. The inherent vibration period of the wing is very short, the elevator is not too sensitive, and damping and longitudinal moment of inertia great enough. By bracing the wing, a relatively thin wing section could be used (Goett. 549) so that the drag of the struts could be neutralised. A simpler construction resulted from making the fuselage six-sided.

As a logical development from the " Professor " came the " Wien " (Fig. 8a). By means of refined design of details the empty weight of the " Professor " was barely exceeded. The further development consisted obviously of increasing

the span, using a higher cambered wing section, a more rounded fuselage cross-section and fared fittings.

Apart from the " Professor " type, the aircraft built for the Munich Club by Dr. Kupper are a departure from the Darmstadt types. This Munich type began in 1928 with the " Kakadu," a cantilever type with a large span, and has been notable in the following years. The wing-form, section and spar design are original. The latter is a thin-walled box spar of plywood of almost rectangular cross-section. This stressed-skin type had only been used up to this time in metal aircraft. As only a few experiments have been carried out over the behaviour of such plywood box spars, and mainly because the buckling strength of the thin skin is not exactly known, this design met with no approval, and is now not even used by Kupper himself.

Fig. 9 shows the three main steps of advancement from the " Vampyr " type. The " Professor-Wien " type was more a result of the experience gained in

Pelzner (1920/21).

Nuernberg seat-type (1921).

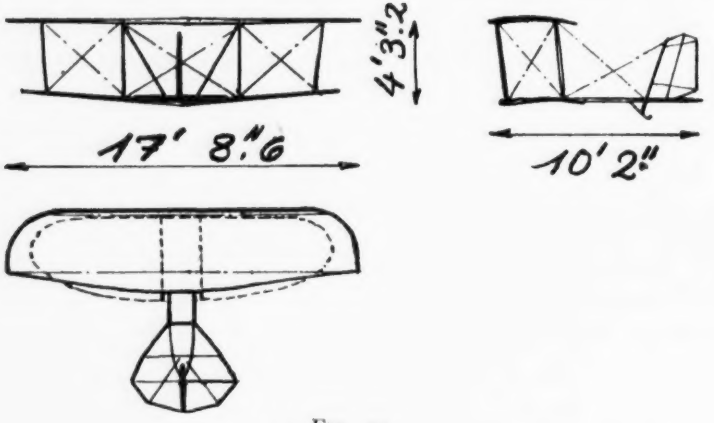


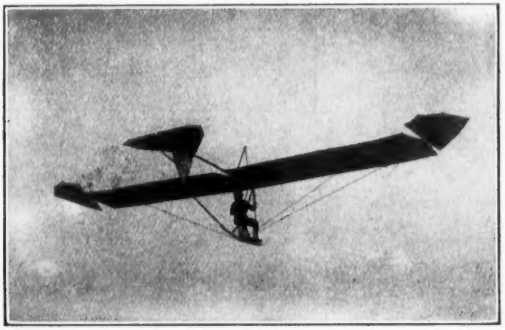
FIG. 10.

Pelzner 1920/21.
Empty weight, 20lbs.
and development to seat-type glider.

the development of training and practice aircraft, of which I shall now give a short resumé.

The first training glider was the suspension type glider which was used by Pelzner in 1920-21 (Fig. 10). It was similar to the Chanute glider. In 1921 the Nuernberger Club developed a small biplane seat-type glider out of the above type, and which became the first training glider. One could hardly say, however, that this machine had any remarkably good characteristics. At this time the biplane was favoured as better flying characteristics were expected from this type. Actually, this machine flew like a parachute. A notable step forwards

München (1921).



F. Schulz (1922).

An early trainer, the Pegasus (1925).

Djävlar annama (1923).

FIG. 11.

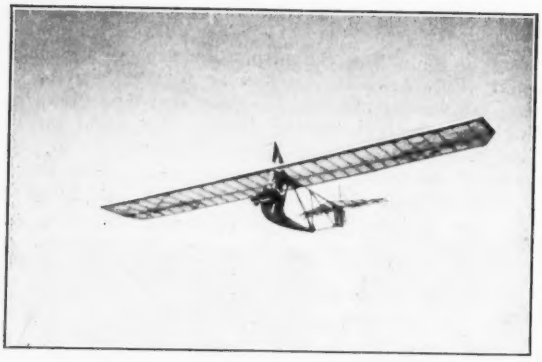
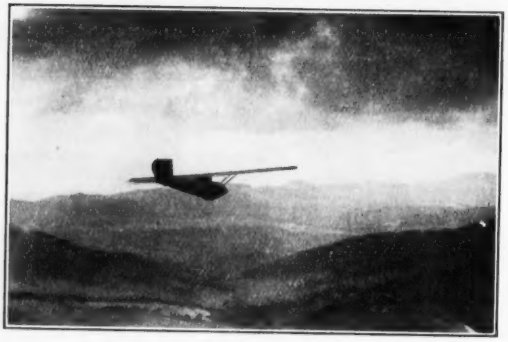


FIG. 12.
Zoegling.



Edith.

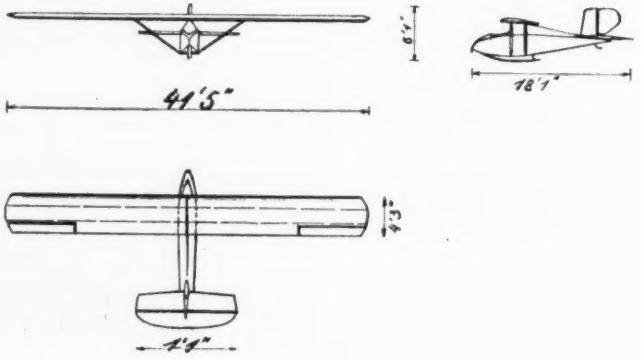


FIG. 13.
Edith, Darmstadt.

was the primary training biplane " Frohe Welt " (Happy World) designed by F. Stamer. The construction was simple and serviceable, the flying characteristics very pleasant, and at the same time the gliding angle and sinking speed were quite suitable for the passing of the glider pilot's examination of the time (present A Licence). There was much trouble caused by damage to the lower wing due to wing-down landings, and later it was also found difficult to pass the tests for B Licence in this machine.

FIG. 13A.
Bremen.

For these reasons the monoplane type came into favour after this time. The development began with the Munich machine of 1921 (Fig. 11). This machine was, however, not designed as a training machine and had a special control system. The structure was merely taken over. Then came the Schulz machine of 1922, which was built of broom-sticks and tin cans, and in which Schulz flew for eight hours. On the basis of this forerunner I built the practice sailplane " Djävlar annama " in 1923 with the idea of building the simplest and

Proefling.

cheapest sailplane possible. If one limits the span one comes, as will be seen later, necessarily to this form. This type was continued from year to year in the slightly changed editions built by Schleicher in Poppenhausen, and has been further in great numbers built as a purely beginner's machine. A notable example of the type concerned is the " Pegasus " of the Martens Flying school, whose simple structure enabled it to be produced very cheaply. From experience

with the " Pegasus " the " Zoegling " (Beginner) (Fig. 12) was developed, and as a further continuation of this series, the Stamer-Lippisch glider, known in England as the R.F.D. " Dagling."

It now remains to discuss the development of the practice sailplanes. The founder of this type was the Darmstadt aircraft " Edith " (Fig. 13) of 1922, which was a braced high-wing type with an almost rectangular wing plan-form. From this type the " Bremen " of the Weltensegler Society was developed. As the next type, I designed the " Hangwind " in 1924, from which I developed the

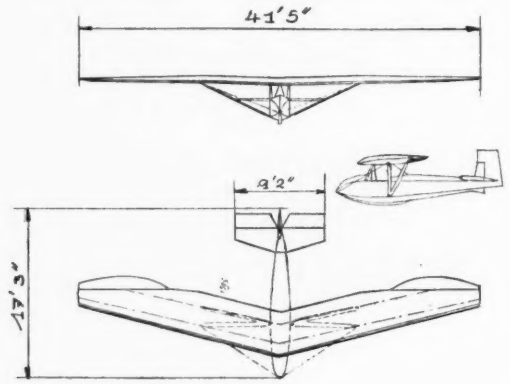


FIG. 14.
Falke (1930).
R.R.G.

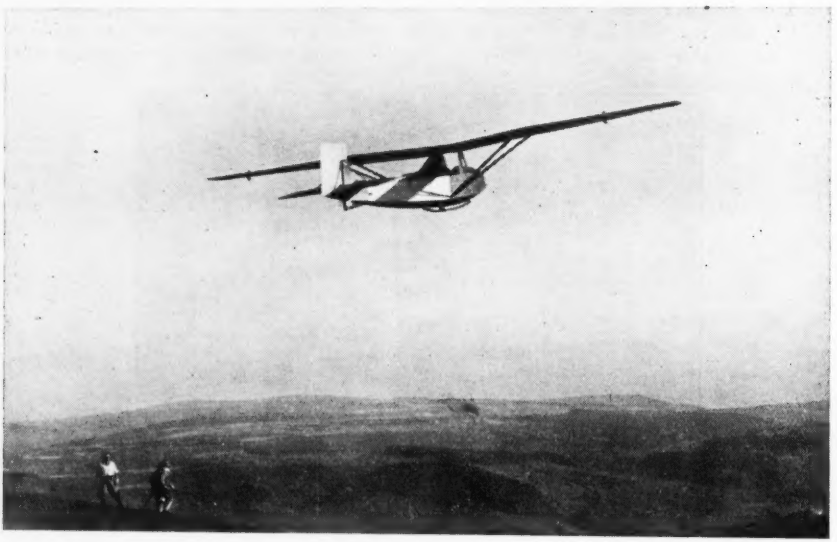


FIG. 14A.
The Falke in flight.

" Pruefling " in 1926. Whereas in the years before, it had not been possible to train pupils who had never been power-pilots as far as the C Licence, the " Pruefling " made this possible, which was very vital for the further development of the gliding and soaring flight movement. The " Pruefling " was improved in 1928 and during the last year replaced by the " Falke " (Falcon) type (Fig. 14). In this design I had in view the production of a machine which combined the greatest possible safety against crashing with the best possible flying qualities. The principles underlying this type will be discussed later.

FIG. 14B.

Besides this development of normal gliders and sailplanes the development of several experimental types also took place, so that already in 1922, for example, several tailless aircraft had been tested. These questions belong, however, already to that province which is connected quite generally with the meaning of gliding and soaring flight for the whole of aviation. In this connection motorless flight is to be the natural pioneer and guide for the future development of aviation.

Part II.

Before I go into the details that determine the carrying out of the design of a glider or sailplane, it is necessary to discuss briefly the physical basis of soaring flight.

There are two possibilities of soaring flight:—

1. Static soaring flight, which depends on the presence of rising air currents.
2. Dynamic soaring flight, which depends on the presence of air currents varying in direction and strength.

Horizontal flight in an upwind takes place when the rising speed of the air is equal to the sinking speed of the aircraft. Therefore the best aircraft is the one which has the least sinking speed. This sinking speed may be derived in the following manner.

We use the symbols:—

W = weight (lbs.).
S = area (sq. ft.).
s = semispan (ft.).
V, V_z, V_x = velocities (ft./sec.).
ρ = density of air (lb. sec.^{2}/ft.^{4}).
k_R = resultant air force coefficient.
k_L = lift coefficient.
k_D = drag coefficient.
ϵ = gliding angle.
A = aspect ratio.
$A = 4s^2/S$.

As you can see by the help of the diagram (Fig. 15), the following conditions must be fulfilled:—

Weight = resultant air force . . . . . (1)
Tan wind direction = drag/lift . . . . . (2)

or expressed in formulæ:—

$$W = \sqrt{(L^2 + D^2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$
$$\tan \epsilon = D/L \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

that is

$$\sqrt{(L^2 + D^2)} = \sqrt{(k^2_L + k^2_D)}\, S\rho V^2 = k_R S\rho V^2$$
$$V^2 = V^2/\sin^2 \epsilon = V^2\,[1 + (k_L/k_D)^2] = V^2\,(k_R/k_D)^2$$

accordingly the sinking speed is

$$V = \sqrt{(W/S\rho)} \,.\, k_D/k_R{}^{1.5} \quad . \quad . \quad . \quad . \quad (3)$$

when one considers that the lift coefficient approximates to the total air force coefficient, and when one assumes normal air density and lets $S = 4s^2/A$, one obtains

$$V_s = 10.3\sqrt{(W/s^2)} \,.\, k_D/k_L{}^{1.5} \,.\, \sqrt{A} \text{ (in ft.-sec. units)}$$

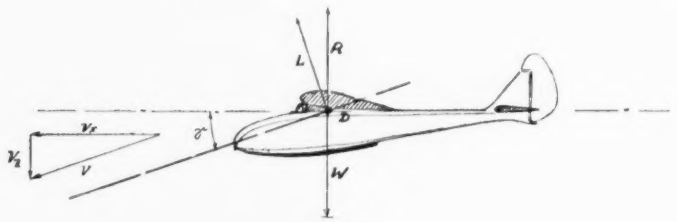


FIG. 15.

The derivation with W/s^2 and A is better, as one can clearly see the direct effect of the span on the sinking speed. We call this function W/s^2 the span loading. In the second factor $(k_D/k_L{}^{1.5})\sqrt{A}$ the aerodynamic characteristics of the aircraft concerned are brought together, and we can calculate this factor for one or several wing sections in dependence upon the parasite drag and A. You will see that one can calculate by this means all the necessary facts for a project or for checking purposes. It must not, however, be assumed as a result of the above elementary details that a low sinking speed is the only measure of the worth of a sailplane. Apart from this the gliding angle and the necessary forward speed at the smallest sinking speed is very important. Generally speaking, one tries to achieve a good range of attitudes with low sinking speed, and at the same time good gliding angles. For one thing, the pilot cannot always be flying at the minimum sinking speed, and on the other side the continual variations in the wind direction and strength are always changing the

attitude of the aircraft. Thus, for the purpose of attaining the smallest possible deviations from the most suitable attitude, quite a large number of other facts must be considered. The most important ones are:—

Stability, especially longitudinal stability.
Manœuvrability, even in unusual flight attitudes.
Sufficient flying speed.

In discussing the aerodynamic basis of sailplane design, I will go into these questions in detail.

Only in the last couple of years in the development of soaring flight has it been possible to make use of the great atmospheric upwind areas. It is thus only too easy to understand that in the times when human soaring flight had not yet been achieved, many phenomena in the natural soaring flight of birds, due to ignorance of these upwind areas, could not be explained as static soaring flight. I would like to make reference here to the extremely interesting explanation of the theory of soaring flight by your countryman F. W. Lanchester in the second part of his Aerodynamics, in which he writes a very thorough discussion of the matter. For this reason the name of that other type of soaring flight, " Dynamical Soaring Flight," originated from Lanchester. This kind of flight is made possible due to the fact that irregularities in the wind without an average upward movement are present. I will show you (Fig. 16), by means of a simple example, how this " dynamic soaring effect " can occur. The bird which flies forwards with the help of wing beats receives the necessary forward thrust through the considerable up and down movements of its wings. The horizontal motion implies, then, that the up and down moving parts of the wings follow a path of wave form relative to the air. If one fixes, as a first approximation, the path of the chief moving wing parts as a sine curve, one can calculate the course and variations of the vertical and horizontal air forces which are caused by such a motion. As you may see from the diagram, when the forces are integrated over a period of one complete oscillation, there is a definite forward thrust and lift.

Let us now assume the air to be in such a motion of oscillation due to friction with the earth's surface or to variously moving air masses. By flying through this air in an aircraft with stationary wings the above mentioned vibration effect would occur. As both forward thrust and lift result, it must be clearly possible to soar in such layers without the help of actual upwind. This effect, which is caused by the periodically changing vertical speed of the wind, is called the " Knoller-Betz effect " and played a great rôle in the first years of soaring flight.

If, for the polar of a given wing, one substitutes a parabolic function of about the form

$$k_D = a k^2_L - b k_L + k_{D0}$$

in which $a = S/4s^2 + \text{const.}$, the increase of the induced and profile drags being considered, one finds that the average coefficient of the horizontal air forces is

$$k_x \sim \tan \phi \, . \, k_L - k_D$$

The curve followed by the wing relative to the air is expressed as

$$Y = m \sin X$$

while the variation of the lift distribution caused by the change in the direction of the air motion along the length of a complete oscillation can be expressed as

$$k_L = k_{Lm} + \Delta k_L \cos X \quad (m \text{ denoting mean value}).$$

One could also give the wing a variable angle of incidence to better the effect. As you see, there is no forward thrust if there is no change in lift along an oscillation, or if one keeps the angle of incidence with respect to the air constant. The maximum forward thrust occurs for

$$\Delta k_L = m/2a$$

and then amounts to

$$k_{X\max} = m^2/8a - k_{Dm}.$$

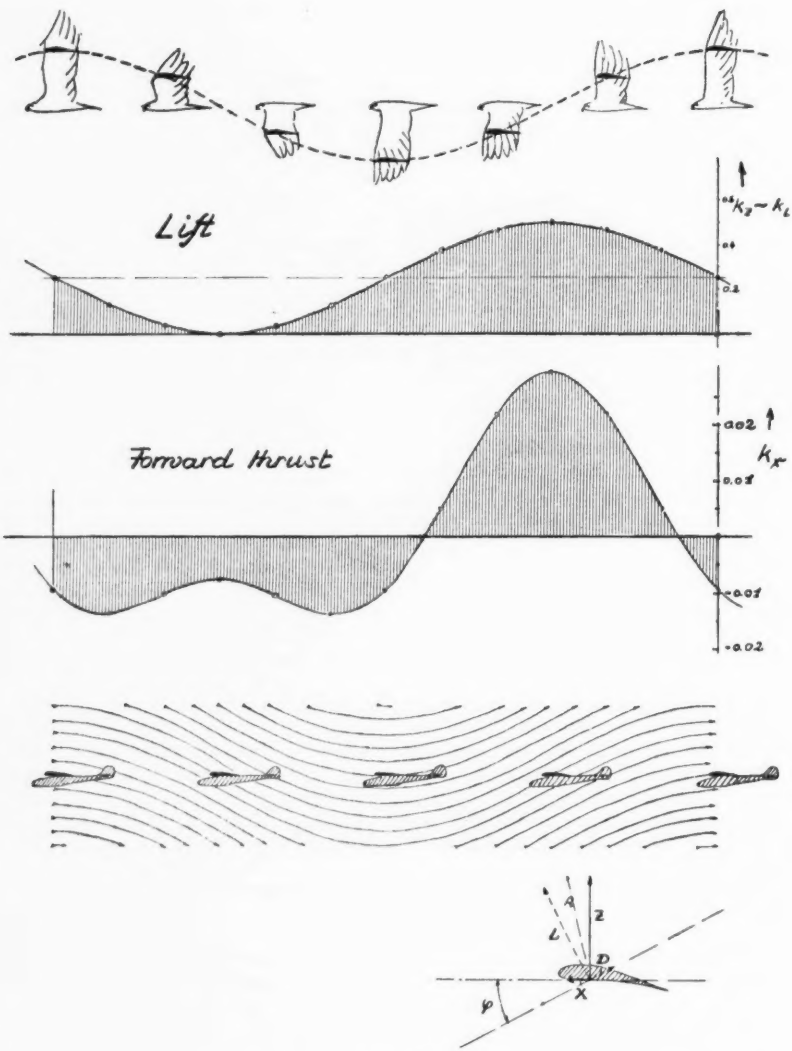


FIG. 16.

Dynamic soaring flight as inversion of flapping flight.

$X = \sin\phi \,.\, L - \cos\phi \,.\, D.$
$Z = \cos\phi \,.\, L + \sin\phi \,.\, D.$
$K_X \doteq t_g\phi \,.\, K_L - K_D.$
$K_Z \doteq K_L.$
$t_g\phi = m \cos X.$
$K_{Lx} = K_{Lm} + \Delta K_L \cos X.$
$K_D \doteq aK_L^2 - bK_L + K_{Do}.$

$$\int_0^{2\pi} K_X = (\Delta K_L/2)\,(m - aK_L) - K_{Dm}.$$

$$K_{Xmax} = m^2/8a - K_{Dm} \Big| \begin{matrix} \Delta K_{Lbest.} \\ = m/2a. \end{matrix}$$

This great variation in lift makes it necessary to have the wing polar as straight as possible. Of course the above considerations are only approximate and disregard acceleration forces.

Even although this effect has never yet occurred to any traceable degree, wind channel tests undertaken by the Vienna Aerodynamic Institute check qualitatively with the above considerations. You see here (Fig. 17) the test

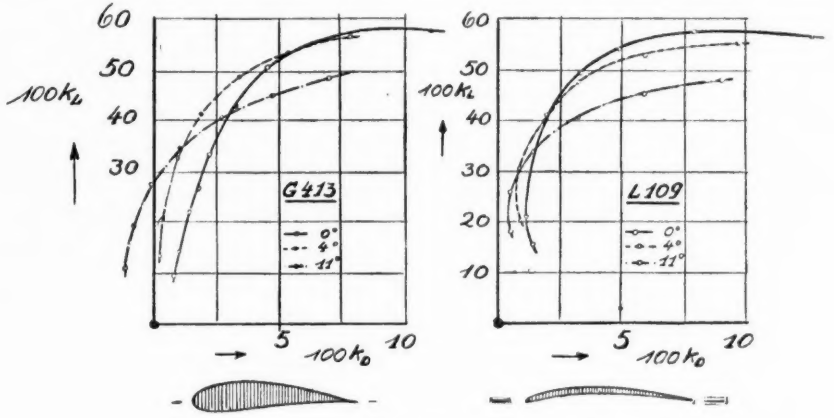


FIG. 17.

The influence of vertical wind oscillation from tests of the Aerodynamic Laboratory, Techn. Hochschule, Wien.

results on two wing models which were tested in a periodically oscillating wind stream.

It is obvious that horizontal oscillations of the wind could cause similar effects. Consider for a moment a wing flexibly mounted on a fuselage. A strengthening of the wind would cause an upward movement of the wing, and by means of the energy collected in the sprung connection, a forward thrust could be exerted in the following lull by means of the downward beat of the wing. The motion of the wing is thus caused by the pulsation of the wind, so one can consider the aircraft as " an aircraft driven through wind pulsations by wing beats." This effect which, as far as I know, was first discussed by W. Birnbaum (Z.F.M., 1924, page 128), reveals a number of matters connected with the performance of cantilever sailplanes of large span in very gusty winds. The superiority of this type over the stiff-winged braced types is very striking. Nevertheless, this effect is very small, and is only of secondary importance as compared to upwinds. In spite of this it is, in my opinion, not wise to ignore this dynamic soaring flight entirely. So long as there exist no incontestable tests carried out with suitable aircraft, it is premature to disregard this kind of soaring flight. There is, unfortunately, in this present lecture no room for a more detailed consideration of this most interesting question, and I think you can read Lanchester if you wish more details.

Part III.

I would now like to consider the aerodynamical and statical bases which determine the design of modern gliders and sailplanes.

The glider which is to be used mainly for elementary training is not supposed to have a high performance. This gives one the opportunity to fit the aircraft

to the purpose of training, and to give first emphasis to safety. It is basically wrong to wish to design such aircraft with the lowest possible sinking speed or especially good gliding angle. One doesn't learn to ride on racehorses!

It is therefore idle to waste many words over the aerodynamic principles underlying the design of elementary gliders.

The choice of a wing section showing a flat curve at high lift coefficients is important so that one can pancake or pull the stick pretty far back in flight (slots are certainly suitable here). It is also important to make all control areas suitably large, through which one must try to combine stability with effective control. Such machines as have insufficient control are unsuitable, for the pupil must know when he has made a false control movement. But I think that last year Mr. Stamer went into these matters with you in great detail.

In the design of a sailplane, as you have already heard, the sinking speed and then the gliding angle are of the foremost importance, and that means that, inside practical limits, we must try to get the best combination of span, empty weight and cost. Of course one starts by trying to find the best possible wing section. When one, after a long search, has actually found a really wonderful wing section, and designed the wing accordingly, one is again in doubt as to which aspect ratio one should choose, and whether or not a change of the wing section would be better in this case, and so on, etc.

The more one works himself into the problem, the more new problems crop up, so that one is finally left to one's discretion. In time such a wandering about is not very satisfying, so that one begins to look for a method which, on the basis of test results and theoretical considerations, would give a general solution of the problem.

We begin with the wing section. As the characteristics of any one wing section cannot give a general view of the problem, in which the probable inaccuracy of single results is to be especially noted, we try to make a systematic determination of values from a large number of tests. We eliminate the induced drag, and then sketch all the profile drag curves together (Fig. 18). Diagrammatically, we get somewhat the following picture: You see that we can substitute a general polar for all the individual polars. The best part of a curve then lies in the neighbourhood of the point of contact with the general polar. Any further calculations we make using the general polar. If by means of this representation we decide on a certain part of the general polar as the most useful in any special case, we choose that section which is tangent at this point of the general polar. We can now substitute the general polar by a suitably chosen function. If, however, we want to achieve a fairly satisfactory agreement, we must use a function of at least the third order, and then the analytical development is very difficult. Therefore we use a mixed graphical analytical method. We have already shown that the sinking speed can be expressed as

$$V_z = 10.3 \sqrt{(W/s^2)} \cdot k_D/k_L^{1.5} \cdot \sqrt{A} \text{ (ft.-sec.)}.$$

And now to determine $(k_D/k_L^{1.5})\sqrt{A}$ generally, we proceed as follows:— The polar of the complete aircraft is replaced by

$$k_{DA} = (1/A\pi)\, k_L^2 + k_{D_\infty} + k_{Dst}$$

in which

$$k_{Dst} = \Sigma k_D S'/S, \quad k_{D_\infty} = f(k_L)$$

then

$$k_{DA}/k_L^{1.5} = (1/A\pi)\, k_L^{0.5} + (k_{D_\infty} + k_{Dst})\, k_L^{-1.5}$$

By differentiating through k_L we get the desired minimum from the following:—

$$k_L^2/2A\pi + (dk_{D_\infty}/dk_L)\, k_L - 1.5\, (k_{D_\infty} + k_{Dst}) = 0$$

So that we obtain the equation

$$A_{best} = k_L^2 / \{ \pi\, (\Phi + 3k_{Dst}) \}$$

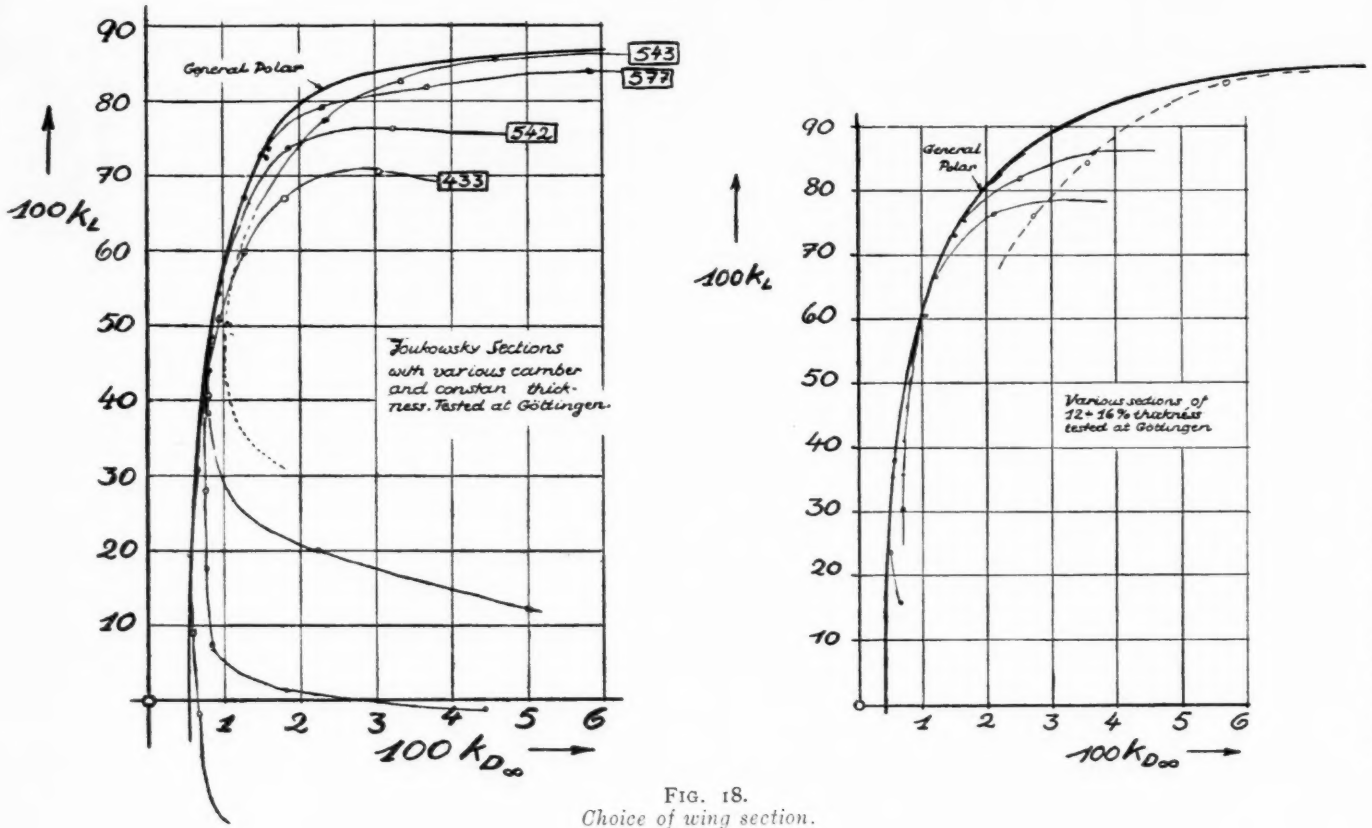


FIG. 18.
Choice of wing section.

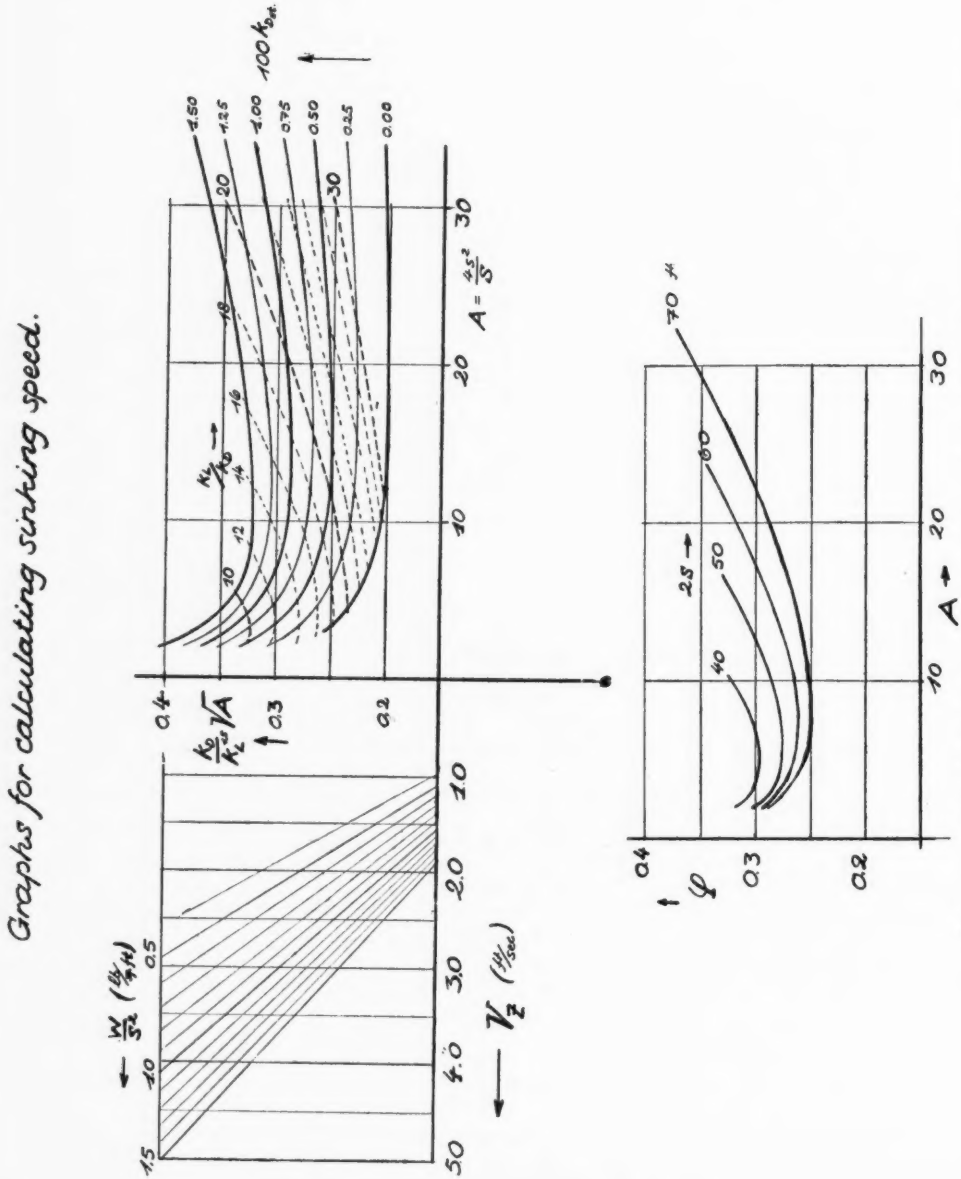


Fig. 19. *Calculated for a structural drag surface of 4.6 sq. ft.*

and then also

$$(k_{D\Delta}/k_L^{1.5})\sqrt{A}=(1/\sqrt{k_L\pi})\{(\Phi+k_{D_x}+4k_{Dst})/\sqrt{\Phi}\}=\phi$$

The expression

$$\Phi=3\{k_{D_x}-(dk_{D_x}/dk_L)(k_L/1.5)\}$$

can be determined for the individual points of the general polar and we can get the aerodynamic sinking speed coefficient $(k_D/k_L^{1.5})\sqrt{A}$ expressed in A and k_{Ds}. In further calculations, for the sake of simplicity, I will designate this expression by ϕ. Here we see the result of such a determination (Fig. 19) in which the results of the Aeronautical Research Institute in Göttingen were used as a basis. In the same manner as for ϕ, one can also fix the lift-drag ratio, and also sketch it on the diagram. Although the use of the general polar is in this case not quite exact (the actual values of the individual sections would be somewhat smaller) it is quite good enough for a check calculation. The shape of the curve $\phi=f(Ak_{Ds})$ shows first that the coefficient ϕ changes only a little when A has a value greater than ten, and secondly, the minima of ϕ lie between $A=10$ and $A=20$. Aspect ratios over 20 are then unfavourable when considering the least possible sinking speed. Merely the improvement of the gliding angle requires a large aspect ratio. And now to be able to judge the conditions for some definite design, we must either decide on the wing area or the span. Then it is still necessary to evaluate the parasite resistance. The existing results in this connection from experiments show much smaller values than those actually shown in flight tests. When one, for example, knows the sinking speed of a sailplane by calculating the same from barographs or test flights, and has further measured the gross flying weight and the span, then ϕ is determined as

$$\phi=V_z/10.3\sqrt{(W/s^2)}$$

If one sets the values found in the diagram over the A concerned, one can determine the value of k_{Ds} and also, of course, the so-called "reduced drag area," or $\Sigma k_D S'$, where k_D is the drag coefficient of the part concerned and S' the area of same normal to the wind direction. I have placed these points for three well-known high performance sailplanes on the diagram, and found as a result that the average value for the reduced drag area is

$$\Sigma K_D S' \text{ (average)}=4.6 \text{ sq. ft.}$$

Even although these values seem very great, and one might assume that the machines concerned were not flown to the best advantage, it would be well to use these values in working out projects, as most aircraft are considerably better on paper than in the air.

To make use of this diagram further. If, for example, we want to build a sailplane with a span of 50 feet, we calculate the necessary wing area and aspect ratio for various k_{Ds} with the help of the above values and $\Sigma k_D S'=4.6$, so that one can place these points on the diagram. From this we can clearly see the effect of change of wing area or of change of aspect ratio. In this connection comes the conclusion that the lift-drag ratio is only noticeably affected by lower aspect ratios. Even from $A=8$ on the change of the lift-drag ratio is very small. This result is also notable and shows the disadvantage of using too large aspect ratios.

In our determination of the sinking speed we have still to estimate the empty weight. For this purpose I have collected a number of actual weight figures, and have attempted to show an analytical connection between this static material (Fig. 20). In accordance with a derivation by Dr. Lachmann, I have let the wing weight be

$$W_s=mS+s^3/n.$$

Further, let the fuselage weight be

$$W_f=k\,.\,s$$

in which, for this approximation, the weight of the empennage is added to that

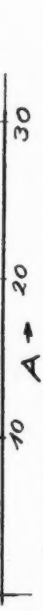


FIG. 19. *Calculated for a structural drag surface of 4.6 sq. ft.*

of the fuselage. The experimental results show that these formulæ can be safely used. The gross flying weight may now be expressed as

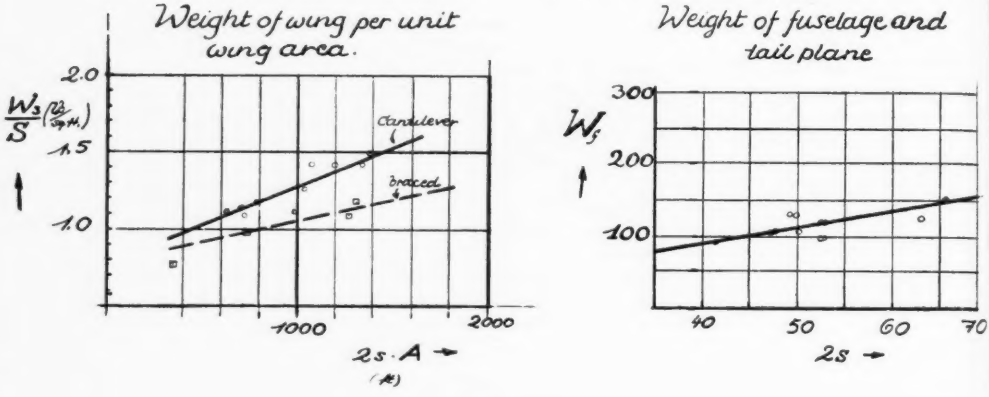
$$W_{total} = m/S + s^3/n + ks + W_o$$

where W_o = weight of pilot and equipment. Now the span loading is

$$W_{tot}/s^2 = m/A + s/n + k/s + W_o/s^2.$$

The coefficient concerned may be expressed as

$m \sim 0.78$ [lb./sq. ft.] $\quad n_c \sim 1000$ [ft.²/lb.] cantilever
$k \sim 4.4$ [lb./ft.] $\quad n_b \sim 1840$ [ft.²/lb.] braced



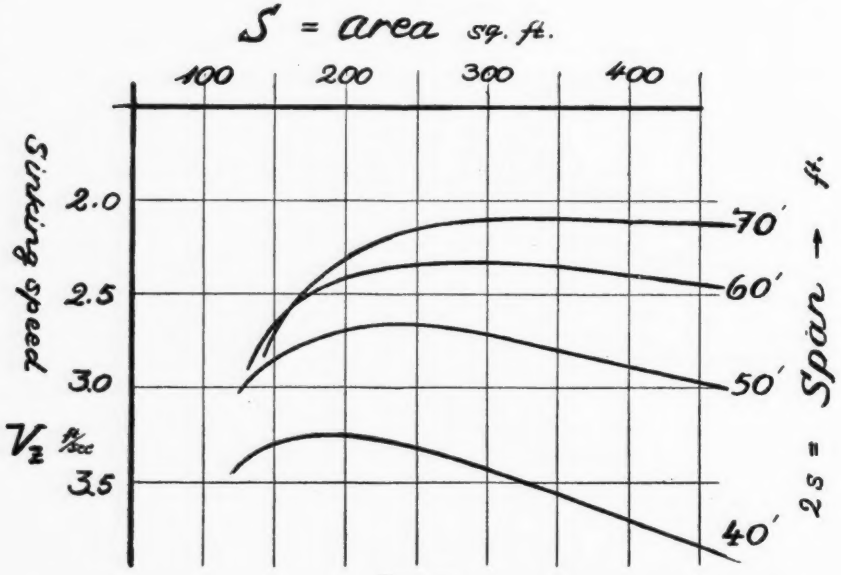


FIG. 20.

Calculated for cantilever sailplanes with reduced structural drag surface of 4.6 sq. ft.

For our design, which we shall consider as being cantilever, we obtain the curve of sinking speed plotted as a function of the area. It can also be seen here that in no case does an extremely large aspect ratio, *i.e.*, small area, give the smallest sinking speed. Although there is no exact relationship expressing the effect of the gliding angle on the " soaring ability," and the general worth of a design, it would seem to be better to use the larger aspect ratios.

If we now go into the design in more detail, we will draw a so-called " speed diagram," using as a basis the now chosen wing section and the calculated values of the final design (Fig. 21).

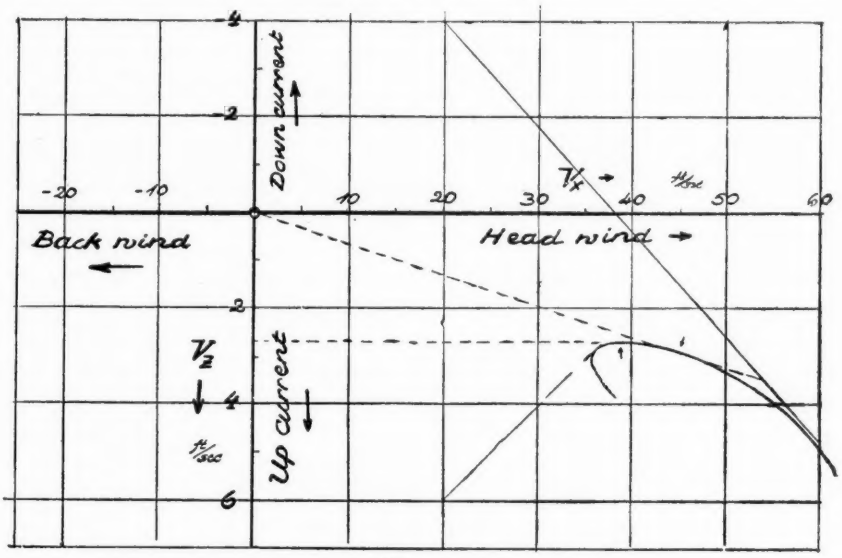


FIG. 21.

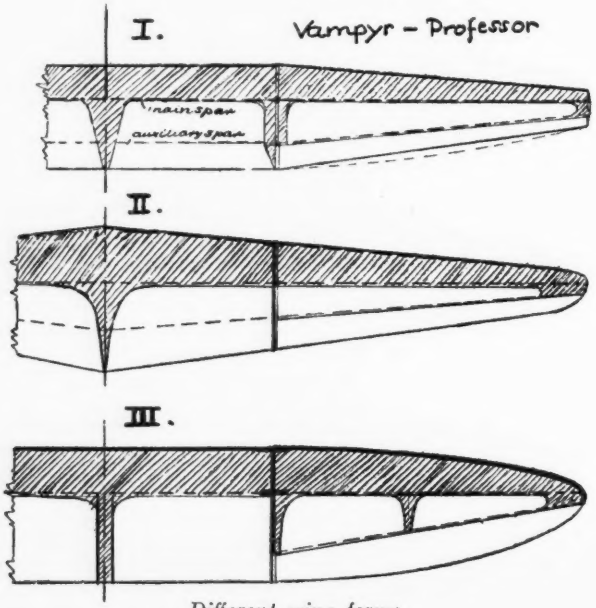
Diagram of velocities.
Calculated for a sailplane of section Göttingen 535.
Span, 50*ft. Area*, 208 *sq. ft. A*=12. *Total weight*, 485*lbs.* (*cantilever*).
Reduced structural drag surface=4.6 *sq. feet.*

One tries to obtain as flat a curve as possible, which can be achieved through the choice of a good aspect ratio. The mean value of the sinking speed, as influenced by changing the air speed, would not vary very much from the best value. Such a diagram is of special importance for pilots who wish to carry out long distance flights in the aircraft. The parts of the flight during which there is no upwind must obviously be so flown that the gliding angle with respect to the earth is as good as possible. Because of the effect of the wind (head or following) this angle is different to that relative to the surrounding air. With the use of the speed diagram the flight attitudes concerned are easily determined, as one fixes the best attitude by lessening the airspeed in case of head wind or increasing it in the case of following wind. One realises that it is correct to fly at higher speed with head wind and at lower speed with following wind, rather than fly at the best gliding angle. If there is upwind or even downwind present, one must subtract it from, or add it to, the sinking speed as the case may be, and then one obtains the best flying attitude by consideration of both effects.

I can unfortunately not go very deeply into the other matters which decide the detail construction of the aircraft. First of all, the designer must always realise that even the machine which is aerodynamically best will be incapable of good performances if the flying qualities are not also good. If the pilot has not perfect trust in his machine and does not know whether she will leave him in the lurch in a difficult situation, it would only be possible under especially good conditions for him to achieve good performances. Because of this it is my opinion that, if it is necessary, it is best to sacrifice a certain amount of aerodynamic efficiency to achieve good flying qualities.

The large spans of sailplanes are naturally only practicable when one uses wings tapered in plan-form. The rectangular plan-form has static and dynamic disadvantages, which cannot be disregarded even for reasons of cheapness of production. This holds true in the same way for the design of engined aircraft, and although there are even to-day many people with the opposite viewpoint, I can only advise them to go to the shipbuilders and tell them that it would really be the best thing for them to build all their frames the same shape and size! The answer that the shipbuilders would give, is also my opinion. Generally one uses plan-forms such as you see sketched here (Fig. 22), of which (I), for example, is for a braced type, (II) is cantilever, and (III) could be either braced or cantilever.

One could base one's choice on the induced drag of these forms which could be worked out by H. Glauert's method. You will, however, find that there is very little difference between the various forms if they do not deviate too much from the elliptical lift distribution. The effect of the plan-form on the manœuvrability is more important. To achieve good control action, one must make sure that in the case of stalling the air-flow breaks away first from the inner portions of the wing. Then one still maintains lateral stability. The shape of the ailerons should guarantee that there is a reasonably great chord right to the outer ends.



Different wing forms.

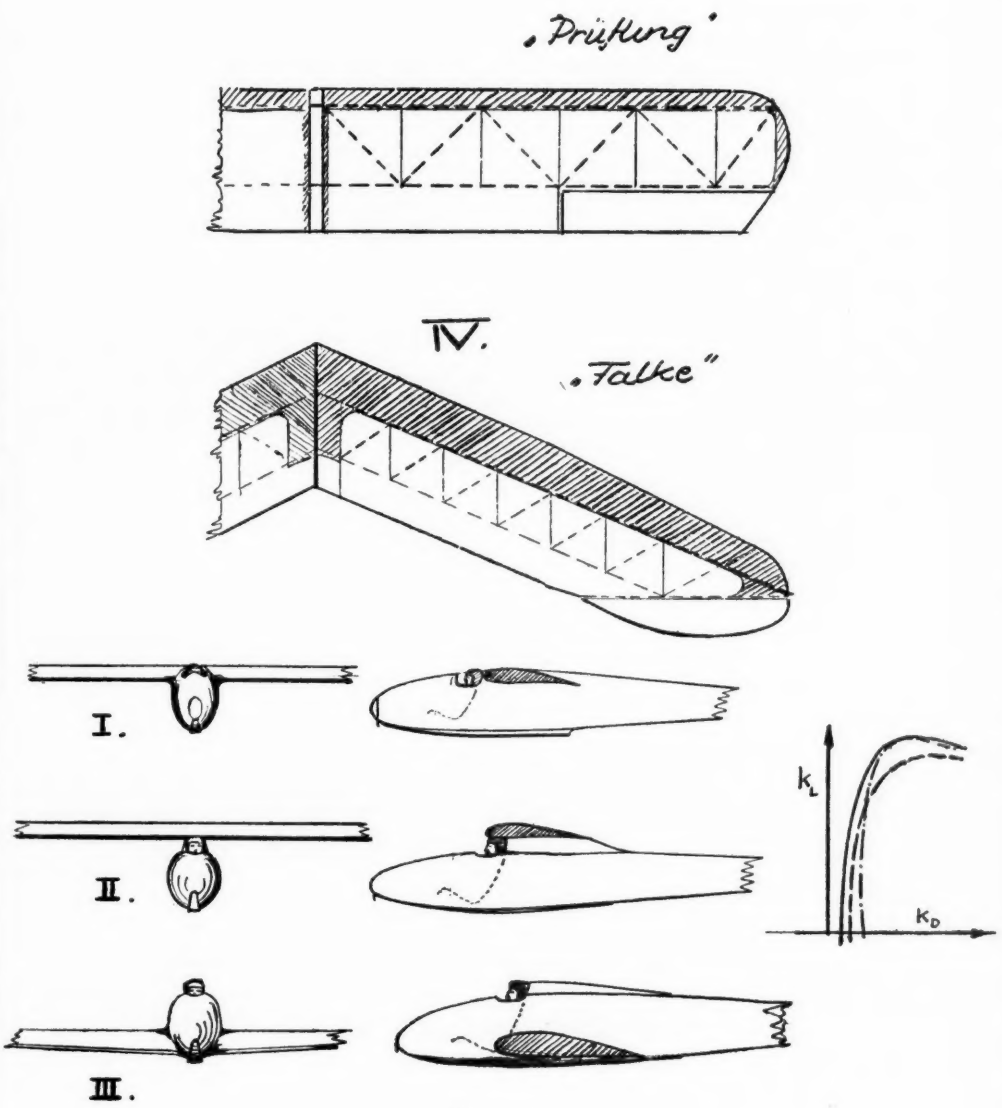
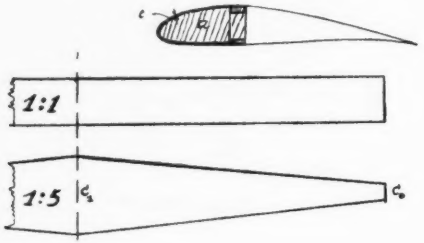


FIG. 22.
Wing fuselage connection.

The shapes shown in dotted lines on the diagram are therefore considerably better. Swept-back and twisted wings have shown very pleasant flying characteristics (IV). Such forms are practically spin-proof. The training sailplane " Falke " which I have designed has confirmed the experience made many years ago. The best position of the fuselage with respect to the wing cannot yet be

laid down definitely. As far as wind channel experiments show, the arrangement (I) is the most suitable. The necessary cut-out in front of the wing makes this arrangement, however, somewhat worse. For this reason type (II) has proven to be better and at present is generally used. The disturbance of the lift over the centre section in soaring flight is very noticeable, because most of the flying is done at a high lift coefficient, and the polar curves are approximately as shown in the diagram. The pure low-wing type has similar disadvantages. While the latter form has often been used successfully in engined aircraft mostly inspired by Junkers, it must not be forgotten that the effect of the slipstream on a wing when the propeller axis is above the wing is more favourable, and that a saving in weight more than balances out a corresponding increase in drag.

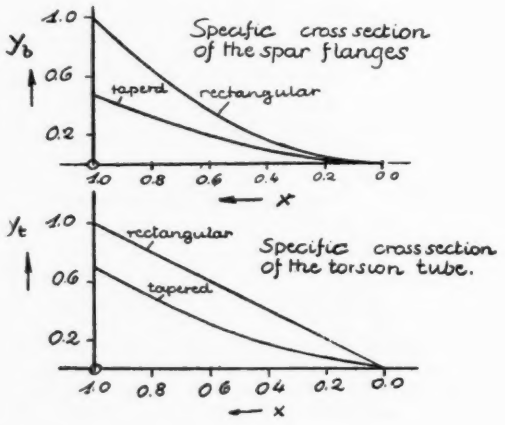
The static construction of sailplanes is to-day almost always of the " Vampyr " type (Fig. 23). One spar at the thickest part of the wing, about 30 to 35 per cent. of the chord from the leading edge takes the bending moments. The forward part of the wing, built up as a box or tube, resists the torsional forces. A lighter secondary spar serves as aileron-attachment spar, and to connect the rib-ends together. The shearing strength of the nose wing-covering can, however, only be guaranteed when the ribs are close enough together to prevent any possibility of the covering buckling. Otherwise folds will appear before the

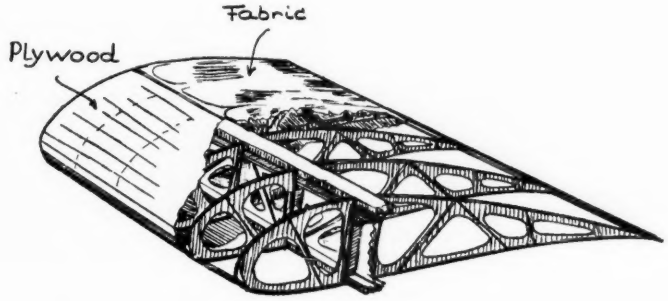


$c_1/c_0 = n.$

$y_b = (x^2/3)\,[3 + x\,(n-1)]/[1 + x\,(n-1)].$

$$y_t = \int_0^x [2/(n+1) + x\,(n-1)/(n+1)]^2/[2/(n+1) + 2x\,(n-1)/(n+1)].$$





Vampyr structure.

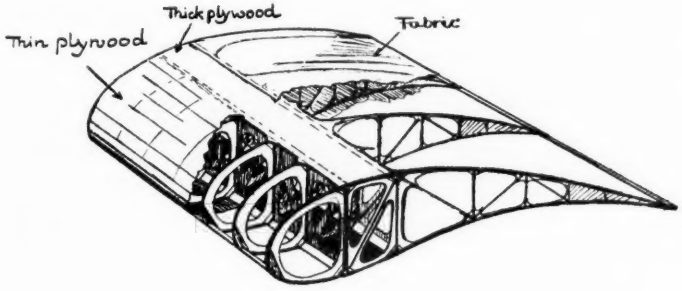


FIG. 23.

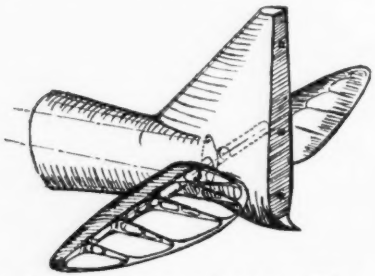
Box spar structure supporting outer covering.

breaking stress is reached. In my experiments I have found that by using plywood attached with grain diagonally over leading edge, the breaking strength is twice as great as normal and the stiffness four to five times as great. A spar built as a thin-walled rectangular box is theoretically best, and is used generally in metal aircraft at present. How to make the thin walls perfectly or suitably stiff with wood construction and also lightly is, however, still a matter for discussion. One is therefore forced to use more material than is necessary, so that the advantage of the construction is only very small. There must be tests carried out in this direction to make the matter clear. A very important matter is the stiffness of the wings to resist bending and torsional vibrations. The latter can be completely overcome when one places the elastic axis of the wing in front of the most forward position of the centre of pressure. Apart from this one must be careful that there is no aileron flutter present.

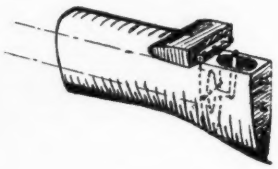
These latter as well as other parts of the outer wings must be very lightly built for this reason. The bending stiffness of this large-span cantilever is in the first place dependent on the height of the spar considered in relation to the length of the cantilever, and also on the load (lift) distribution. Thus in every respect the tapered wing is superior to the wing of rectangular plan-form. The same can be said for the torsion resisting structural members. I show you here a comparison between two wings of the same span and area, one with a rectangular and the other with a tapered trapezoidal plan-form. Assuming the same wing-section and angle of incidence, figures to represent the two flange-cross-sections and the " torsion noses " were calculated. You see that in this case by the use of tapered wings one would save 50 per cent. in weight in the cases

of the nose covering and the spar flanges, over the rectangular wing. It is further clear that for the same loading, the bending and twisting is much greater for the rectangular wing than for the other. For large spans, the most favourable as far as weight is concerned, is the braced (Professor-Wien) type. The extra drag of the struts is almost neutralised by the use of a thinner wing section. This type is only reasonable when torsion and bending can be taken up by the struts, as the torsionally stressed parts are at least as heavy as the parts stressed in bending. We can, of course, not exactly say what is the very best type of wing. Nevertheless, calculations which take into consideration both the aerodynamic and statical points of view are very informative. All these considerations are not only applicable to sailplane design. As far as design is concerned the construction of sailplanes is really a natural experiment plant for all problems connected with power aircraft.

Fuselages of high-class sailplanes are nowadays always of round or elliptical cross-section, and covered with plywood (Fig. 24). The three-longeron type with



Empennage Type Professor-Wien.



Type Darmstadt.

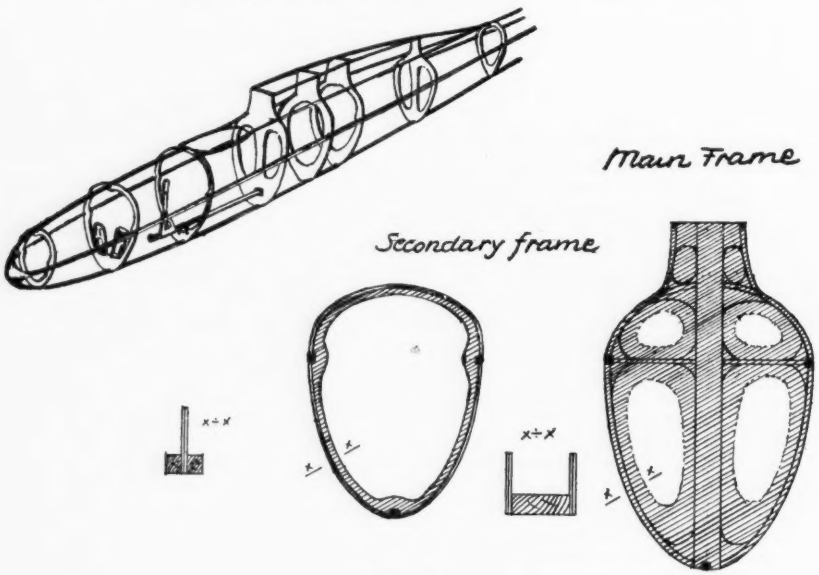


FIG. 24.
Fuselage framework of high performance sailplanes.

a rounded edge underneath is simple to construct and fits the shape of the sitting man very well. In smaller training aircraft the fuselages are four or six-sided structures. Fuselages of extremely small cross-section permit, of course, only of the use of wheel control. For high-performance sailplanes this control is therefore generally used. The control surfaces are designed in the same manner as the wing. In high-performance sailplanes the pendulum type of elevator is generally used. The construction of the fuselage end like that of the " Professor " or " Wien " is usual. In the case of primary training aircraft the regulations state that the horizontal control surfaces must consist of a fixed tail-plane and elevator. There have been no accidents caused by the use of the pendulum type elevator. One must only choose the gear ratio correctly.

In Germany the following regulations guide the strength calculations for gliders and sailplanes.

Group : WING.

Loading case I. Stress corresponding to flight with most forward position of centre of pressure. Factor of safety, 6.

Loading case II. Stress corresponding to flight with maximum torsional load. Factor of safety, 1.

Loading case III. Stress corresponding to a landing (wing weight as load) 6-8 safety factor.

Group : FUSELAGE.

Loading case I. Stress due to load on empennage. Breaking load of empennage is breaking load of fuselage.

Loading case II. Stress by landing. The wing loads are 6-8 times breaking load.

Loading case III. Stressing of wing-fuselage connection by landing on wing. Breaking load 110 lb. applied at wing tip in direction of wing chord.

Group : EMPENNAGE AND AILERONS.

Elevator and rudder : Breaking load 31 lb./sq. ft.

Ailerons : Breaking load 16 lb./sq. ft.

The polar diagram of the wing and of the complete aircraft must be used in the strength calculations.

A proof of the static stability is also necessary.

You will perhaps be surprised that we have only taken two cases of loading into account in the strength calculations for the wing. But I do not see what use a point between the most forward position of the centre of pressure and diving

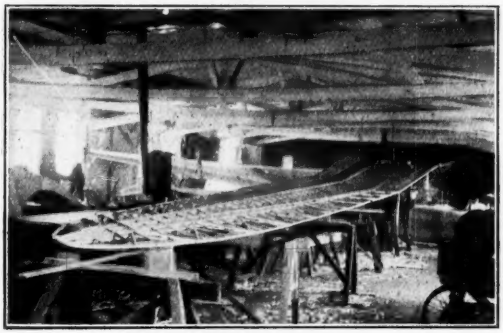


FIG. 25.
Wing construction, " Fafnir."

flight would be. The consideration of case (I) proves whether there is enough strength to take care of bending and forward thrust. The consideration of case (II) proves whether there is enough strength to take care of torsion and backward pressure. These cases take care of the extreme forces. Years of experience have shown that the above strength demands are enough. Apart from the above the same methods and bases may be applied and used in sailplane design as are used in power aircraft design. The many detail questions, which would doubtless be very interesting would, I fear, take us too far from our subject.

By means of several photographs (Figs. 25-33) I would like to show you constructional details of several types.

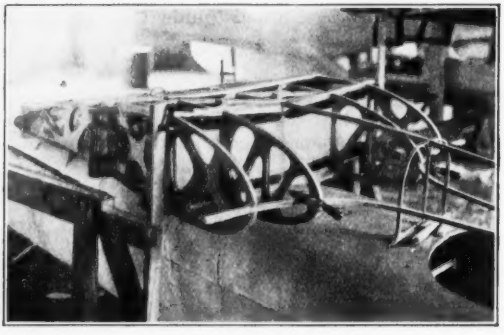


FIG. 26.
Wing fuselage connection and centre section with fittings, "Fafnir."

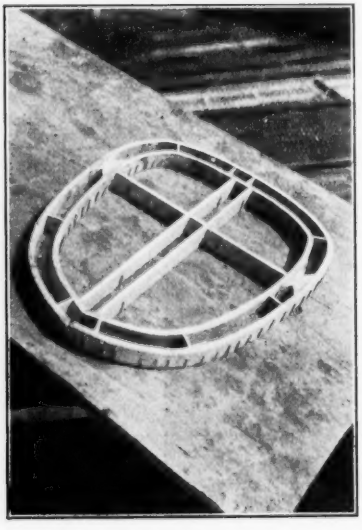


FIG. 27.
Fuselage frame.

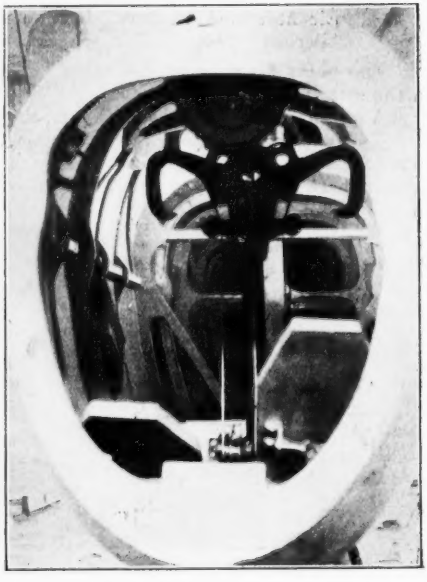


FIG. 28.
View of pilot's seat and controls, "Fafnir."

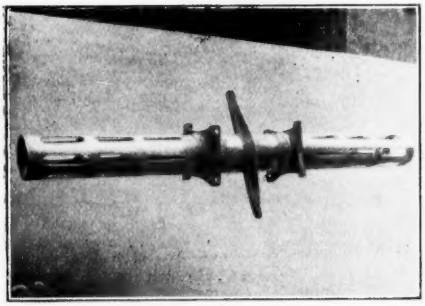


FIG. 29.
Elevator hinge tube, "Professor."

FIG. 30.
Empennage, "Luftikus."

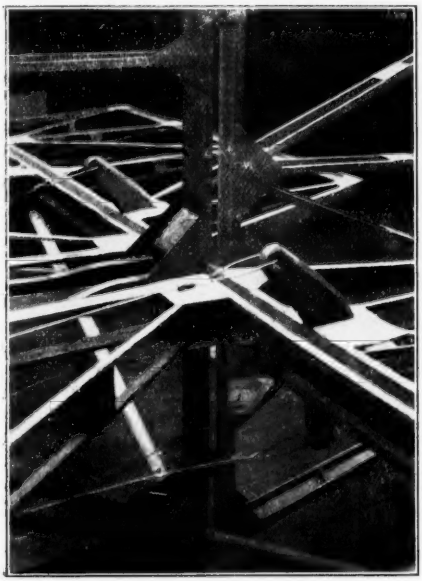


FIG. 31.
Fuselage construction, "Pruefling."

FIG. 32.
Uncovered empennage, "Pruefling" two-seater.

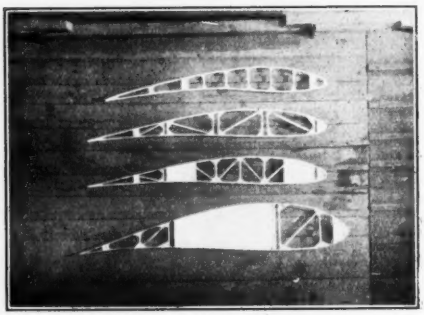


FIG. 33.
Wing ribs.

The Importance of Motorless Flight to the Development of Aviation

I would now like to finally consider the meaning of motorless flight for power-driven aircraft.

Last year Mr. Stamer told you about the value of gliding as flying training. Therefore I really do not need to repeat that the pilot who has learned to master a light sailplane in wind and clouds, has gathered experience for his whole flying career which could never be so clearly taught in a power-aircraft flying school. He must have a thorough knowledge of weather observation and meteorology if he wishes to make any good performance. Soaring flight has brought new knowledge to aerometeorology, and the new researches on the vertical movement in the atmosphere have at least been stimulated by soaring flight.

The technical significance of sailplane design at the present time is made clear by the foregoing parts of my lecture. On this account I would like to consider merely one specially fruitful sphere of motorless flight research.

Since the end of the war (1914-18) one has begun to use aircraft for commercial purposes all over the world. This air traffic has grown from year to year and to-day covers wide districts of various continents. There is air traffic

everywhere except where its proper field of activity is, that is there is no air traffic *between* continents where rapid communication is really necessary, and where air traffic could actually " fly by itself " or pay its way. Now I ask you, what is the use of this fastest of all means of communication, if it has no use in its proper field of activity? And what is the sense of this much-advertised speed when it is only procured by a phenomenal waste of energy? I cannot help but think that the main problem of this means of communication, that is, the economics of it, is carelessly handled.

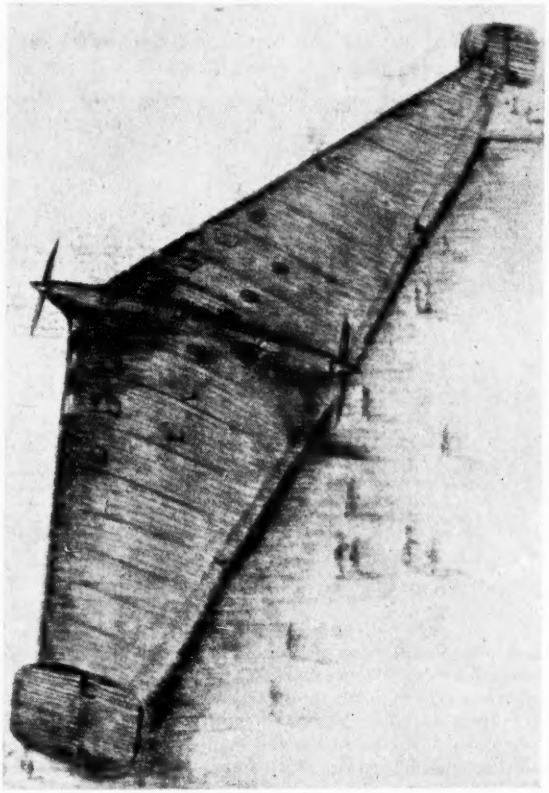


FIG 34.
The Ultimate.
Herr Lippisch's all-wing passenger-carrying machine.

What would you think of anyone who equipped a farmer's waggon with a 100 horse-power engine and drove from petrol station to petrol station at sixty miles an hour? The most sensible thing to do would be to advise him to use ball-bearings to cut down the frictional resistance of his vehicle. Perhaps you laugh at all this, but you must admit that most of our present-day aircraft have not as good a frictional coefficient as our original farmer's waggon (without ball-bearings!) Or perhaps you don't believe that the waggon would roll down a grade of 1 : 15 by itself? But perhaps you can tell me of an aeroplane which has a gliding angle as good as 1 : 15? The only aircraft which can compete with

this honourable rival are a couple of high-performance sailplanes which struggle for the first prizes every year on the Rhoen. The comparison shows you where we must start to develop economical aircraft. The energetic reduction of the frictional coefficient is a vital necessity in aircraft design. One could, of course, make these tests in a wind channel and that is the usual thing to do at present. One soon notices that the wing alone has always a better gliding angle than the whole aircraft with fuselage and empennage. Then it would be clearly the best thing to do, if one tried to build an "all-wing-aircraft." But why don't people build such aircraft? Quite obviously because one sees no possibility of safely developing the aircraft from the wind channel state to full size. There is one perfectly safe and proved method and that is called motorless flying.

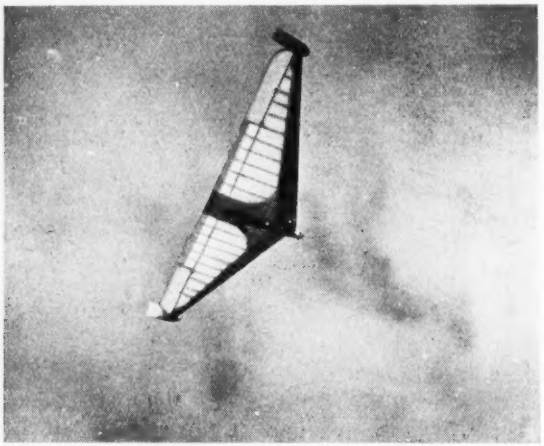


FIG. 35.
The first stage: a model in flight.

This method was actually known since the beginning of aviation, and it has only been given up to-day because, on the basis of long experimentation and experience, the behaviour of a normally built aircraft is well enough known that the controllability and stability of a new type can be guaranteed before the first flight. But how a quite new type of aircraft would behave is beyond our knowledge, and even calculated characteristics with the help of wind channel tests cannot be guaranteed. The method which we have used with good success for some years, I will now explain by means of a shortly sketched example.

We will now for example create an "all-wing-aircraft," and have worked out a suitable project (Fig. 34) on a basis of various theoretical and design considerations. Before we start designing a man-carrying aircraft, we build a model of such dimensions that we can use the laws of dynamic similarity to advantage. To do this, it is necessary, as is well known, that the Reynolds numbers that appear in flight must exceed the critical region between laminar and turbulent friction, and also that the ratio of the wing-loading between model and full-size aircraft must be to scale. We test this model in free flight, notice the effects of various control settings, the behaviour in flight in very gusty winds, and in short, everything that is included in dynamic and static stability. By changing the model we correct any possible deficiencies, and experiment until we are satisfied with the flying properties of the model. (Model 35.)

Now we can go a step further and venture into the design of a man-carrying glider. This aircraft will be so built, for practical purposes, that after a successful test an engine may be installed. Perhaps you might ask why we don't use a small engine in the first place? Mainly so that we can remove all possibilities of danger, as far as we are able.

The first short flight over flat land in the glider is absolutely safe, whereas the first flight with a power-aircraft cannot be so, as an insignificant error can have a crash and fire as a result. Even if the glider did crash, it would not be dangerous because of the low speed. But it is quite a feat to crash from a height of a few inches. By a progressive choice of ground with steeper slopes we gather experience on the behaviour of the new aircraft. We study the effect of the various controls, the effects of outside disturbances, and can always fall

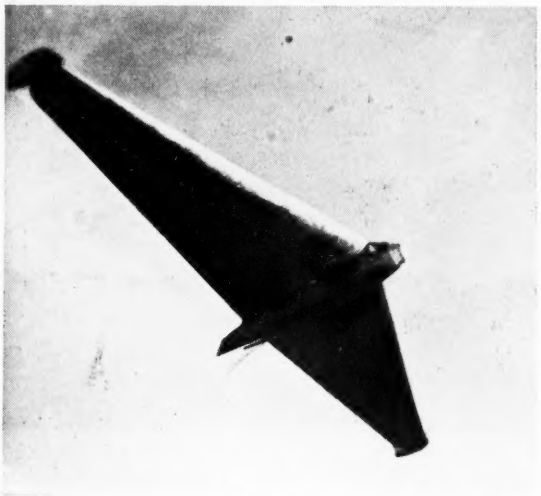


FIG. 36.
The second stage; the full-size two-seat glider.

back on the model tests in difficult cases. Finally, we are so far that we can carry out longer gliding and soaring flights with the aircraft. I hardly need to point out that by this means one can gather sufficient experience so that trials with an engine installation may be begun. If one still has doubts as to the operation of the airscrew drive, one can use tests with free-flying motor-driven models, as an aid to the solution. If one hasn't a large landing field at one's disposal it is better if possible to use the skid (instead of wheel undercarriage) for the first flights with the engine. The final steps are certainly not at all necessary to mention.

The successful experiments with tailless aircraft which have also been carried out in this country by Captain G. Hill, are the beginnings of a new development period in aviation. I realise that against the arguments which have been put forward, people have an army of doubts in reserve, and most of them wonder whether the longitudinal stability (dynamic and static) is sufficient in the tailless type, and consider that the structural weight of swept-back wings would be greater than that of a corresponding normal wing, and that the manœuvrability is not sufficient. The researches which I have carried out have proved that these views are wrong because the original assumptions were quite different. Un-

fortunately because of the lack of time I cannot consider these questions in detail, but hope, however, that the discussion will lead in this direction.

I would like to stress once more that motorless flight and the researches stimulated by motorless flight are not only of use in the province of the gliding and soaring flight movement. The value of this new branch of aviation lies in the broadening of our knowledge in scientific, technical, and practical flying fields, and those whose vocation is concerned with the success of aviation cannot help occupying themselves seriously with the gliding and soaring movement.

The meaning and purpose of my lecture would be fulfilled if I have succeeded in giving new knowledge to those who are already interested in these things and have won over, as future collaborators, those who have been looking on.

FIG. 37.
Herr Lippisch's tailless glider does its trials at the Wasserkuppe.

DISCUSSION

The CHAIRMAN: He expressed, on behalf of all present, the most sincere thanks to Herr Lippisch for his very interesting lecture, and said that obviously he had spared no trouble in the preparation of it.

The Chairman also expressed his extreme interest in the wing spoilers which had been referred to, and asked for more information concerning them.

Mr. E. C. GORDON ENGLAND (*Associate Fellow*) (Chairman of the British Gliding Association): He thanked the Royal Aeronautical Society for its unfailing co-operation with the Association in all matters in which they were jointly interested, and said it was due to the Society that the members of both bodies had had the benefit of hearing a lecture by Herr Lippisch.

He would not discuss at length the technical aspects of the problem, because he was interested in propaganda and, indeed, had almost given up his life to propaganda on behalf of the gliding movement. The lecture, he prophesied, would do much to further the art and science of gliding, and in the future would be looked back upon as marking a new stage of progress. He expressed regret that only a few technicians from the aircraft manufacturing firms were present at the meeting, and was bold enough to suggest that perhaps they imagined they had learned all there was to know about their own problems, and were

under the mistaken impression that gliding had not a great deal to teach them. Herr Lippisch had indicated, however, the ways in which gliding could greatly increase their knowledge.

A point with regard to which there was a great deal of variance of opinion as between those interested in gliding and those engaged in the manufacture of aircraft was that of the value of a true and highly finished surface as a factor in the efficiency of a soaring machine. He asked if Herr Lippisch set a high value upon a highly finished skin surface.

The work of Herr Lippisch and others justified the faith of the pioneers of aviation to a very marked extent. Mr. Gordon England expressed the opinion that in the course of time—and not a very long time—it would be proved that the late Joseph Weiss (under whose auspices he himself had been introduced to aviation) had made a very remarkable contribution to the science of aviation, and one which had not yet been fully appreciated. Like many pioneers, Joseph Weiss had lived before his time.

Having regard to the indication by Herr Lippisch of the value of the swept-back wing, he asked if Herr Lippisch believed in the theory that there was some wave motion in the air which called for the use of a swept-back wing or a wing of distinct plan-formation, and that that plan-formation was greatly influenced by the air speed of the machine; in other words, at varying speeds a varying plan-formation was necessary, to say nothing of the cross-section of the wing.

He would like to thank the Rhon-Rossitten Gesellschaft for having, with great courtesy, and in the most reasonable way from the financial point of view, placed at the disposal of the British Gliding Association the complete working drawings and specifications of the " Falke " machine. These were available to clubs and private constructors throughout the country, so that any who wished to test the merits of Herr Lippisch's latest intermediate machine had only to apply to the Association for the necessary information.

Mr. LOWE WYLDE: He felt that, in view of the experience of Herr Lippisch, one could only listen to and thank him for having given the meeting the benefit of his experience in a concentrated form. Frankly, as one who appreciated the work that was being done in Germany, and as one who was very desirous that we in this country should emulate it, he felt somewhat afraid. In view of the work carried out in Germany during the last ten years, he felt that they had before them a very great task; it would take ten years to get things fitted up and to gather the amount of knowledge that their German colleagues had already gathered. He joined in the hope that as the result of the meeting there would be a full realisation in this country of the fact that this work in connection with gliding was helping in the perfection of motor-driven aircraft, and that with the greater interest and support which should be forthcoming we should be able to carry out further investigations on behalf of aviation.

Captain LATIMER NEEDHAM (*Fellow*): Referring to wing loading, he said that for the case in which the centre of pressure was in the most forward position the factor given in the paper was 6, but he believed that many of the German sailplanes were built to factors of 9, 10, and even higher. He asked if Herr Lippisch would discuss that matter further. With regard to the ply covering over the leading edge, which was supposed to take the torsion, he said that, since it passed over and under the main spar, it must be subjected to the maximum bending stress to which the spar was subjected. Therefore, it had to take that stress before it could take any stress due to the torsion. He would appreciate the lecturer's comments on this point. With regard to the elevator and rudder, the loading given for the limiting velocity nose dive was 31lbs. per sq. ft. That appeared to be unduly high, but undoubtedly Herr Lippisch would be able to explain the reason for that.

Furthermore, by fixing the tail load in lbs. per sq. ft. instead of a load of so many lbs. spread over the whole tail, designers might be inclined to cut down the size of the tailplanes.

Captain G. T. R. HILL (*Fellow*): He emphasised three important points in design which had been made in the course of the lecture. The first was that, in the opinion of Herr Lippisch, tapered wings were greatly superior to parallel wings when they came to the big aspect ratios; secondly, that there were great virtues, from the point of view of stability, in the swept-back wing; and thirdly, it was clear, from one of the diagrams—which he hoped would be reproduced in the Journal—that the strutted wing was definitely considerably lighter than the pure cantilever wing. Those statements were all supported by a detailed mass of evidence, which was very valuable, and in that respect they contrasted favourably with the usual sweeping assertions, unsupported by evidence. The fact that the lecture included much detailed data made it an extremely valuable contribution to the Society's Proceedings. When visiting Herr Lippisch a few weeks ago he had seen on his table papers, journals and technical information of all sorts and in various languages, and was glad to say that copies of the JOURNAL OF THE ROYAL AERONAUTICAL SOCIETY were prominent amongst them.

Captain Hill asked if Herr Lippisch would be good enough to elaborate his remarks on the bending stiffness of the wings. Trouble had been experienced in some machines, he said, due to the wings being too flexible in bending, resulting in what he believed was called a " personal oscillation "—a longitudinal oscillation, as distinct from a lateral oscillation. The pilot was unable to distinguish between the movements of the machine due to bending of the wings and those due to wind gusts, and in trying to correct the motion, he actually forced an oscillation, which on one occasion had had disastrous results.

In emphasising the concluding paragraphs of the lecture, Captain Hill recalled that recently, when travelling home from Germany by air—in an aeroplane which he would refer to as the " A " type—he had seen through the cabin window something dear to the textbook writer, a rectangular flat plate, placed at 90° to the wind stream. The plate was about 1ft. square, and after puzzling for some time over its object, he had realised that it was a means of blanketing off the oil cooler in cold weather in order to keep the oil from becoming cooled too much. If that sort of thing could be done on modern air liners, he said, one felt that the message contained in the concluding part of the lecture could, with great advantage in certain quarters, be very fully digested.

Captain ENTWISTLE: He was particularly interested in the reference by Herr Lippisch to dynamic soaring flight, and asked whether, in the full-scale tests which had been carried out in Germany, any quantitative measurements had been made of the lift obtained in the conditions referred to and also what maximum values had been obtained in soaring flights made in a definite upward current. He agreed with Herr Lippisch that the effects experienced as a result of variations in the strength of a horizontal wind current were negligible in comparison with the large-scale effects which were due to large masses of rising air, but he also agreed that the former might ultimately prove to be of practical value, particularly in relation to glider design. He also asked whether Herr Lippisch could give any information in regard to the instruments which had been found in Germany to be most useful from the point of view of the measurement of the magnitude of the upward currents experienced in soaring flight.

Squadron Leader ENGLAND (*Associate Fellow*): In most of the sailplanes illustrated, he noted that the fixed tailplane had been dispensed with, and would like Herr Lippisch to state whether the movable tailplane had found favour in the sailplane, whereas it was of little use on power-driven machines.

He would like to be enlightened as to why the controls on the " Zoegling " machine, which was definitely intended for training were, with the exception

of the elevator, inferior. Surely on a training machine it would appear undesirable to have a powerful elevator without good directional and lateral control.

Mr. DAGNALL: He would like to know whether Herr Lippisch recommended a dihedral or a flat angle.

Dr. LACHMANN (*Associate Fellow*): The lecture was a compilation of the essentials of sailplane science, and those who were concerned with this science in all countries would be very grateful to Herr Lippisch for having done that work. Some of the formulæ which Herr Lippisch had given had been guarded for some time as holy secrets by the various gliding corporations. Every aircraft designer must admire the refinements which had been achieved and must also envy the sailplane designer the possibilities which the sailplane afforded of the application of aerodynamics to such fine limits. It was easy, of course, to blame the poor aircraft designer for bad aerodynamics in the design of his machines, but there were a good deal more practical limitations. Assuming that a highly efficient sailplane had a span loading of about 10lbs. per sq. ft., in order to achieve the same result in the latest Handley Page 42-seater it would be necessary to have a wing span of 3,000ft. Everyone concerned with practical aircraft design knew that there were specifications which did not always allow of the proper application of aerodynamic refinements. Dr. Lachmann asked what were the bending deflections measured in the air in bumpy weather on the high-span sailplanes.

Commenting upon the influence of sailplane design upon motor aircraft, he said it was not by accident that the designer of one of the most efficient German commercial planes, Messerschmidt, was formerly a designer of sailplanes, and he had incorporated in his latest designs all the characteristic features of the highly successful sailplanes, *i.e.*, large span, tapered wings and single spar construction. These machines were considered to be highly efficient; he recalled that when Messerschmidt had commenced to apply his sailplane experience to commercial aircraft, many people in Germany had been very doubtful as to the outcome of it, but ultimately he had been successful. Dr. Lachmann expressed the belief that monoplanes of large span with tapered wings and single spar construction would be the future type for commercial purposes.

Mr. J. H. PAYNE: It seemed that in the sailplane wings of the mono-spar construction there were no drag forces of any sort. He understood that the main spar bore the lift forces and that the tube at the front bore the torque, and he asked if the main spar also bore the drag forces on the wing.

Mr. C. T. CUSS: Speaking as Chairman of the Wiltshire Light Aeroplane and Glider Club, and not as a technical authority on the design of gliders, he suggested that danger might arise through the partial success of any one type of glider having undue influence on other types. Having observed that in 1922 and 1929 the champion gliders of the world had come to grief, it had occurred to him that careful research was necessary with a view to reducing the resistance to gliding arising from the use of the usual types of ailerons, elevators and rudders. He recalled that about ten years ago he had had the good fortune to shoot a 5-cwt. porbeagle shark off the North Devon coast. The exquisite streamline speed form of it had made a most lasting impression on his mind. He asked Herr Lippisch if he considered that there were possibilities of reducing resistances and eddying from ailerons, elevators and rudder by using a flexible fuselage, wings and tail, all inflated and operated by pneumatic controls, which would take a bump without damage, and better than the very delicate ply-wood. He pointed out that the vertebra of the shark (of which he exhibited a model) had no bones in it; it was just cartilage and skin. He asked if there were prospects of developing a flexible fuselage on similar lines, *i.e.*, having a vertebra through the centre and an inflated fuselage around it. If that were so he would suggest that it might be possible, by pneumatic controls, gradually to curve the fuselage in any desired direction, still maintaining the streamline effect and in

that way reducing resistance. There would also be a movable snout worked by controls, to assist turning. He also mentioned the fact that the shark referred to was covered with a slime, as the result of which resistance was reduced, and he asked what importance Herr Lippisch would attach to that fact. By giving the least shudder the shark could slip away at speed—and, of course, it was dependent upon speed for its existence.

Mr. HOWARD-FLANDERS (*Associate Fellow*): He asked Herr Lippisch what was the torsional deflection of the spar in, say 25ft., of the large span sailplanes in flight. If a single spar was used, it was fairly obvious that the torsional resistance of the tip of the wing would be less than that at the root when in flight, and he would like to know what was the allowable deflection.

Mr. ASHWELL-COOKE (*Companion*): He asked for the opinion of Herr Lippisch on the use of slots on high efficiency sailplanes. Also, speaking as one who was connected with the London Club, which owned no less than three machines of Herr Lippisch's design, he asked for information concerning any system that might be in use in Germany to ensure adequate maintenance of club-operated machines.

Mr. W. O. MANNING (*Fellow*): Referring to one of the diagrams which had been exhibited, he said it appeared that three-ply was shown as being used for the web of a spar. He felt sure that, on a spar of a power plane, three-ply would not be used in compression; possibly there was something slightly misleading in the drawing. With regard to gliding angles, and the suggestion that a farm waggon would roll down a gradient of 1 : 15 by itself, so that it had the equivalent of a gliding angle of 1 : 15, Mr. Manning said he believed there had been several aircraft built in this country having a gliding angle slightly better than that. He mentioned the matter in order to show that the aircraft designers in this country had not been altogether unmindful of the importance of reducing resistance. As well-known examples, he mentioned some of the machines used for the Schneider Cup contest, in which the reduction of head resistance had been carried out quite effectively.

The high aspect ratios used could not be used in power-driven aircraft, not only because of the increase of weight but also because of the lack of rigidity in the wing development, and on these lines must therefore be confined to gliders.

Colonel the MASTER OF SEMPILL (Past-President of the Royal Aeronautical Society) (*Fellow*): Those who were interested in motorless flying had been asking for some time past for information which Herr Lippisch's paper had now supplied. As showing how much the visit of Herr Lippisch to this country was appreciated from the scientific point of view, he mentioned that the Aeronautical Research Committee—the first Committee of its kind to have been set up in any country—had invited Herr Lippisch to attend its meeting on February 3, and to give it the benefit of his advice. Further, Herr Lippisch was to visit Yeovil and had kindly consented to address that branch of the Royal Aeronautical Society on Monday, February 2. Anyone who had to grapple with the preparation of a lecture would realise the enormous amount of work involved, but if one had to prepare a lecture in one language, translate it into another, and convert all the figures of another standard of measurement, and prepare special slides, the labour involved would certainly be trebled. It would be impossible to find suitable words to express our gratitude; they must, in fact, confess that they were ashamed of themselves as they realised that they had no one capable of dealing with this subject in a foreign language in the very clear way that the lecturer has done. Colonel the Master of Sempill also mentioned that Herr Lippisch had been greatly assisted in the preparation of the lecture by a Student Member of the Royal Aeronautical Society, Mr. B. S. Shenstone, who was studying at the Wasserkuppe in the Research Section under the direction of Herr Lippisch. He (the Master of Sempill) asked Herr Lippisch, on behalf of the Royal Aeronautical Society, the British Gliding Association, and all those

interested in motorless flying in this country to take back to the Rhon Rossitten Gesellschaft, and particularly to the President (Professor Georgii), the very cordial thanks for the considerable help afforded in the past and proffered for the future.

The vote of thanks was accorded with acclamation.

Air Commodore J. A. CHAMIER (*Associate Fellow*) (*communicated*): With reference to Herr Lippisch's lecture, he would like to ask if he has ever tried wings with flexible trailing edges?

He states in his lecture that in gusty winds there is a marked difference in performance between cantilever sail planes, of which the wings were somewhat flexible, as compared with the stiff-winged braced type. He attributes this to the dynamic soaring effect and the beating of the air by flexibility in bending of the wings.

Is it not possible that the single spar wing has a considerably more flexible trailing edge than the double spar braced wing, and that it is to the superior efficiency of the flexible trailing edge that the improvement may be attributed?

He also wished to ask whether Herr Lippisch could tell them to what extent gliding has confirmed wind tunnel results? It is in this check of wind tunnel results, unhampered by such things as the influence of slipstream of propellers, that to his mind the greatest scientific value of gliding lies. The lecturer, he understood, designed some of the Göttingen wing sections, so it is almost certain that he must have been interested to make comparisons between free gliding and the wind tunnel.

Mr. SCOTT-HALL (*Associate Fellow*) (*communicated*): While at the Wasserkuppe he was singularly impressed by the magnitude of the deflections which take place at the wing tips of some of these very high aspect ratio sailplanes under gusty conditions. On at least one machine the wing tips could be seen with the naked eye deflecting when the aircraft was flying at close quarters, and indeed he was informed that this inherent " springing " of the structure made the aircraft very comfortable to fly. Naturally the first thoughts that this sight produced were upon the likelihood of dangerous flutter conditions arising. Has serious trouble of this kind ever been experienced? While on the same subject, he was very interested in the flutter " acceptance " test carried out on all aircraft competing in the annual R.R.G. competitions, in which the wing tip is deflected by hand (the aircraft being stationary on the ground) and the oscillations per minute counted. He understood that if the oscillations were less than 120 per minute the aircraft was limited to flying in light winds. Is this figure correct?

Herr Lippisch mentions in his paper the good anti-spin effects of sweep-back and twist on wings. What experience has he had with spinning on the ordinary types? It would seem very easy to get into a spin on one of these craft while cloud soaring, and he was given to understand at the Wasserkuppe that once these aircraft reach the stall they spin suddenly, due to the high lift wing sections commonly used. Has any difficulty ever been experienced in recovery?

REPLY TO DISCUSSION

In answer to the Chairman.—The wing-spoilers were tested on the two-seater of the Dresden Academical Flying Club, being inspired by wind channel tests made in Holland. The Dresden machine had 60 feet span and the wing-spoilers were each about one foot long. The effect of the spoilers was very satisfactory. By their use, the normal (almost elliptical) lift distribution is disturbed, and the induced drag at high angles of attack very much increased. It should be noted, however, that the actual lift is affected to only a very small degree.

Mr. Gordon England.—A highly polished wing surface is of the utmost importance for the maximum lift. Even a dusty wing is noticeably inferior at

low speeds. This is not only important in soaring machines, but also in power aircraft, especially when stretching a glide to make a forced landing field. Observations of bird flight in Nature uphold Mr. England's views on plan form. In the case of the vulture, at low speeds it has a swept-forward plan form with a lift distribution fuller than elliptical. When flying normally, the plan form is practically straight and the lift distribution elliptical. In the case of high-speed flight, the plan form has sweepback and the lift distribution is flatter than elliptical, which is not unfavourable, as the induced drag in the high-speed flight case is of no importance. The main reason, however, for sweepback is for the sake of increased stability and manœuvrability.

Captain Latimer Needham.—Although only a factor of safety of 6 is demanded in the C.P forward condition, it usually occurs that in a carefully-worked-out design the final factor is considerably greater. In the case of school machines, the writer considers that a higher factor of safety is necessary, not because of air evolutions, but because of ground evolutions. Regarding the question about spar bending and the torsion nose, it must be remembered that plywood has a lower modulus of elasticity than the normal wood of the spar underneath. Therefore the plywood layer is not so highly stressed in bending as would at first be assumed. Of course the torsion nose can take up a certain amount of bending. It should be remembered, however, that in the case of maximum bending, the torsion is very small, and in the case of maximum torsion the bending is almost zero. The high tail loading of 3½lb. per sq. ft. is necessary because of the high terminal speed that can be reached by high performance sailplanes, because of their very low drag. The size of the tailplane is not fixed by the load but by the stability calculations, which are also demanded by the Technical Commission as one of the airworthiness conditions.

Captain Hill.—The longitudinal oscillations of the aircraft caused by the wing oscillations can certainly become very serious. Thus all wings which have a period on the ground in still air of less than 120 per minute, are given only a " limited " soaring certificate.

Captain Entwistle.—There have been no quantitative measurements made on dynamic soaring flight at full scale. Such measurements are especially difficult, as the structure of the air itself is very difficult to determine. As far as instruments are concerned, the sailplane is the best instrument for measuring upwinds when equipped with meteorgraph, accelerometer and variometer, and airspeed indicator. The sinking speed of the sailplane is determined by barograph readings taken on a long glide out of the influence of upwinds after the sailplane has been towed to a great height (say 6,000 to 10,000ft.). Also the use of weighted pilot balloons has been very satisfactory.

Squadron Leader T. England.—The movable tailplanes are much lighter, and with suitable gearing down are not too sensitive. The fixed tailplane is used in powered aircraft because the flying requirements demand that the aircraft fly " hands off." Regarding the " Zoegling," due to the very low C.G. and low speed, it is very difficult to make the ailerons and rudder sensitive. However, the most important control for the *ab initio* student is the elevator, and this has been made sensitive.

Mr. Dagnall.—A flat angle is recommended.

Dr. Lachmann.—There is considerable difference between the performance calculations for sailplanes and powered aircraft. A simple consideration leads to the conclusion that for a sailplane, sinking speed is of most importance. Sinking speed depends on " span squared loading " $= W^{\frac{1}{2}}/2s$. For an engined aircraft, the power required is of first importance, *i.e.*, weight times sinking speed is the measure of quality, or power required is proportional to $W^{1\frac{1}{2}}/2s$. As the span affects the weight of the aircraft considerably, it is easy to understand that the " span squared loading " of powered aircraft must be higher than for sailplanes.

In spite of this, I am of the opinion that a decrease of the span loading in modern aircraft would give better performance, which is shown in practice in the Messerschmidt commercial monoplanes.

Mr. J. H. Payne.—Under normal flying conditions, no drag is caused by the wing relative to the chord axis, in sailing and gliding. The torsion nose takes up the thrusts under other conditions.

Mr. C. T. Cuss.—No considerable decrease in drag is to be expected by using a flexible fuselage. The control surfaces could, however, be improved by flexibility and this was also tried during the first years of soaring flight. Due to practical and technical difficulties, the further use of flexible control surfaces was given up. In my opinion, the same difficulties would occur with attempts to use pneumatic flexible fuselage and wings. Even although there might be an increase in safety, the pilot would probably get heart failure, due to the strenuous exercise necessitated by having to pump up every puncture or blow-out, or make a forced landing. But joking aside, it may be stated that there have already been patent applications for wings and fuselages of this type. Although it is admitted that the slimy coat certainly lessens the water resistance of a body, the most important matter in decreasing air resistance is to make the surface as smooth and clean as possible.

Mr. Howard Flanders.—In normal flight the torsional deflection of the spars of high performance cantilever sailplanes is usually not more than 1 or 2 degrees at the wing tips. Although in engined aircraft a limit of $3\frac{1}{2}$ degrees is given, there are no limits with sailplanes, for it would be impossible to build a wing of reasonable weight having an angle of torsional deflection as small as $3\frac{1}{2}$ degrees in the diving flight condition. On the other hand, this deflection is not dangerous and has a stabilising effect if the torsional axis lies forward of the centre of pressure.

Mr. Ashwell Cook.—From my point of view, the addition of slots to a high performance sailplane would be of little value, for in spite of the most careful design they would probably add a certain amount of parasite drag, and increase the inertia of wings which are already rather heavy. However, flight tests with slots are to be carried out at our Institute in order to test them thoroughly on sailplanes. This has been made possible by the very friendly co-operation of Handley Page, Limited. In Germany there exists the system of construction inspectors which are licensed by the R.R.G., and almost all clubs have such an inspector among the members. The inspector is responsible for all repairs and airworthiness. In the absence of the inspector, the manager of flying of the club takes control.

Mr. Manning.—I am very glad to hear that several English aircraft have gliding angles of over 1 : 15. The very careful design of the Schneider Trophy machines has awakened the admiration of the world, and the lecturer counts himself as among the admirers. The Schneider Trophy machines give a practical proof of the effect of streamlining on performance, and this streamlining should be applied also to normal aircraft with equal care. Even to-day there are many designers who do not seem to realise that a number of details not carefully carried out when added together make a very large drag. It must not be thought that these remarks are meant to be especially applicable to English aircraft, as they are applicable to the aircraft of all countries. The drawing of the box spar of the " Kakadu " shows three-ply used as spar web, as the Munich group were not able to make special plywood out of veneer with mostly longitudinal grain. This matter of producing a special plywood was one of the reasons why this structural method, as mentioned in the lecture, was not continued. If such a special plywood could be suitably produced, thin-walled spars such as those used by Rohrbach could be successfully built of wood. Regarding the matter of the high aspect ratios, the answer to Dr. Lachmann may be referred to.

Air Commodore Chamier.—Although the lecturer has not built wings with flexible trailing edges, he agrees with Air Commodore Chamier regarding the value of the flexible trailing edge. Regarding the question on flight and wind tunnel tests, these fulfil quite different purposes. The free flight tests of one-third to one-quarter scale models were carried out more as stability tests than for any other reason, and no measurements were made. Free flight testing of stability of some of the rather radical types considered at the Wasserkuppe was of great importance, as even the most careful calculations need practical checks.

Mr. Scott Hall.—The figure of 120 per minute is correct. The springing of the cantilever wings certainly aids the machines in soaring, having a wing-beat effect, causing a certain amount of dynamic soaring. Of course the springing helps to wash out the effect of bumps. In the early days there was a certain amount of trouble due to flutter, but this has been successfully overcome by the flutter acceptance test. In the case of ordinary large span sailplanes in which the whole wing stalls at approximately the same angle, a steep spiral commences easily and often develops into a spin. They come out of the spin normally. The only thing to note in practice is not to use too much bank at low speeds, as the great span makes the lift on the upper wing increase rapidly, and so the bank increases, developing into the spiral. At such times the aileron control will not bring the machine out of the bank. The tendency of the machine must be followed, and the machine brought out of the dive in the ordinary manner after sufficient speed is gained.

Colonel the Master of Sempill.—I thank the Master of Sempill for his extremely kind words which he has expressed on behalf of the R.Ae.S. and the B.G.A. I will not deny that the preparation of the lecture was rather difficult. But these difficulties were very small compared to the recognition which motorless flight has won by means of the lecture. From my point of view, it was to be taken for granted that the lecture must include everything of importance which has been achieved in the last ten years' development, and that these things be handed over complete without hesitation. I believe that it is obvious, that among friends one must always be open and helpful, as it is only on this basis of mutual effort and working that anything of value may be brought forth.

BRANCH LECTURES

THE PRIMARY IMPORTANCE OF MECHANICAL TESTING IN AIRCRAFT CONSTRUCTION*

BY

I. J. GERARD, M.Sc., Assoc.M.Inst.C.E.

(*Lecture read before the Bristol Branch, December 18th, 1930, and the Yeovil Branch, March 26th, 1931.*)

Introduction

An accurate knowledge of the strength of materials is more essential in aircraft construction than in any other branch of engineering, as every aircraft component has to be reduced in weight to a minimum compatible with safety. Equally essential in aircraft construction is the use of every available refinement in calculation, although it is not possible as yet to estimate with sufficient accuracy from calculations alone the strength of many of the components. At every stage in aircraft construction, mechanical tests are therefore made, first to determine the qualities of the materials, then to verify the strength of each part and finally to check the strength of the complete machine.

Owing to the wide range of my subject and to the limited time available, I propose to deal briefly with some of the aspects of mechanical testing of special importance in aircraft research and development, describing more particularly some of the test apparatus and methods of testing I have had opportunities of developing.

Part I. Tests of Materials

Tensile Tests.—The most widely accepted criterion of the mechanical strength of the vast majority of constructional materials is undoubtedly the tensile test. For metals used in aircraft construction, this test is usually made in a standard testing machine provided with tension shackles of a type which ensure axial loading of an accurately machined test piece.

Where the material to be tested is exceptionally brittle, it is found necessary to use axial loading shackles of the type first introduced by Professor Robertson. These shackles enable satisfactory results to be obtained from brittle materials where other types of axial loading shackles have failed. Most aircraft material is, however, fairly ductile and it is found in practice that axial loading shackles comprising universal pin joints give satisfactory results for all materials to standard specifications. The two types of shackle are illustrated in Fig. 1. In the Robertson type of shackle there is only rolling friction, whereas in the universal pin joint type of shackle there is rubbing friction. It may at first sight appear that the second type of shackle would result in poor alignment and consequent bending of the test piece, but actually the test piece in this case takes up its true alignment when the tensile load is very small and retains that alignment throughout the test, provided the testing machine is in reasonably good condition. Comparative tests, using both types of shackle, show that all but exceptionally brittle materials give the same test results for both types of shackle.

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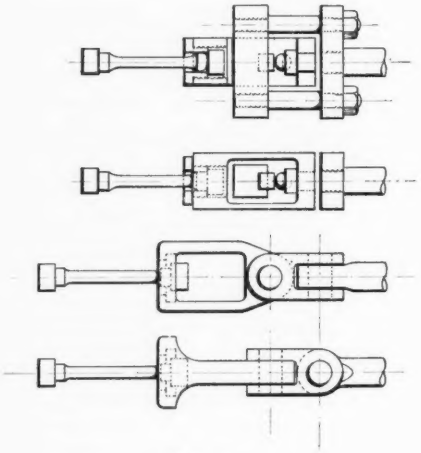


FIG. 1.

Some standard types of test piece in common use at the R.A.E. are shown in Fig. 2.

The properties most commonly sought from the tensile tests of aircraft materials are:—the limit of proportionality stress, *i.e.*, the stress at which the stress-strain diagram ceases to be a straight line, the 0.1 per cent. proof stress, *i.e.*, the stress where the material extends 0.1 per cent. plastically, the 0.5 per cent. proof stress, the ultimate tensile strength, the percentage ultimate elonga-

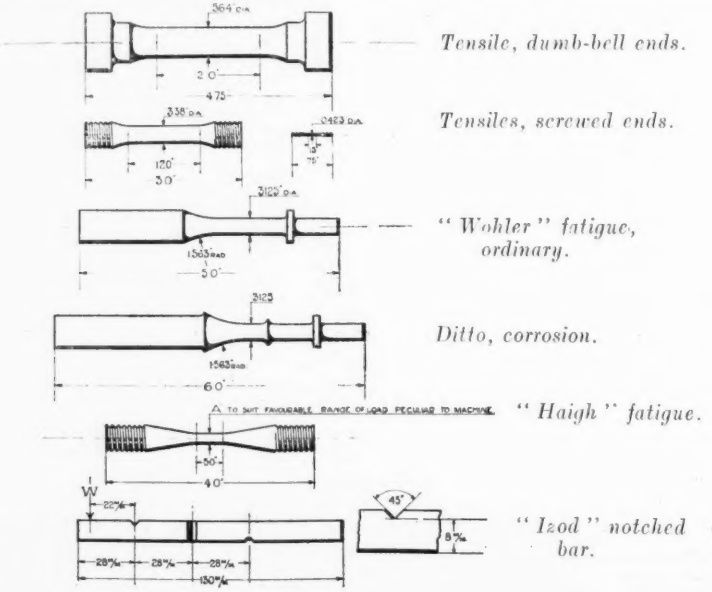


FIG. 2.

tion on a standard gauge length, such as four times the root of the area of cross section for turned test pieces and, say, 2in. for other types of test pieces, the percentage ultimate contraction of area, and the modulus of longitudinal extensibility or Young's Modulus. The yield point is omitted from the foregoing properties as it is marked only in wrought iron and mild steel. In order to make those determinations which involve the measurement of elastic extensions, some form of extensometer is necessary. The extensometer which has been found most convenient for general use at the Royal Aircraft Establishment is

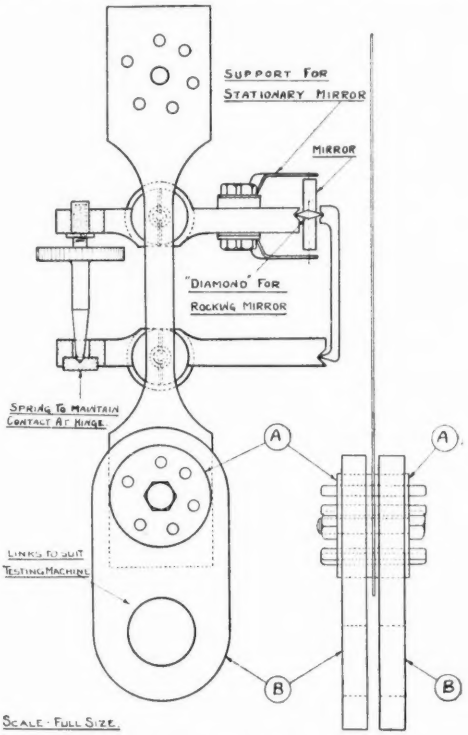

SUPPORT FOR STATIONARY MIRROR
MIRROR
"DIAMOND" FOR ROCKING MIRROR
SPRING TO MAINTAIN CONTACT AT HINGE
A
A
LINKS TO SUIT TESTING MACHINE
B
B
SCALE - FULL SIZE.
TENSION TEST PIECE FOR SHEET METAL SHOWING EXTENSOMETER IN PLACE & METHOD OF MOUNTING IN MACHINE.

FIG. 3.

that originally introduced by Professor Robertson. This is somewhat similar to Morrow's extensometer in principle and is illustrated diagramatically in Fig. 3, and to scale in Fig. 4.

An extensometer of this sensitivity and accuracy, *i.e.*, one reading extensions to one-millionth of an inch, is essential for most research work on the mechanical properties of materials, but for inspectional work, such as that done at aircraft contractors' works, a less sensitive extensometer such as those developed by Major Wylie and by the author, is less costly in the first place, quicker and more convenient in use, and gives sufficiently accurate results.

The order of importance in which other mechanical tests of materials may be placed varies with the use to which these materials are to be put.

Fatigue Tests.—Most engine parts and those airframe parts which vibrate in service are subject to alternating, pulsating or fluctuating stresses and their fracture by fatigue may be caused by repeated cycles of direct, flexural, torsional or combined stresses. It is necessary to know if these parts will have a long life under these conditions. There are two methods of determining this: (1) by applying static and dynamic loads to a sample part in a manner which conforms as closely as possible with the expected conditions of loading when the part is put into service, and (2) by calculating the stresses occurring in service and finding the fatigue resistance of the material by standard tests. The first method may be ideal because it is the more direct method and because the effect of the geometrical proportions of the part is included in the test, but for preliminary design purposes, expense precludes the application of this method to all but a few exceptional parts. In the case of engine parts, type trials serve as the final check in this direction. Certain parts of an airframe, such as ribs, are usually subjected to type vibration tests before adoption. The application of the second method is aided by recent advances in analytical and experimental methods of determining fatigue stresses such as, for example, stresses due to torsional vibration in engine crankshafts

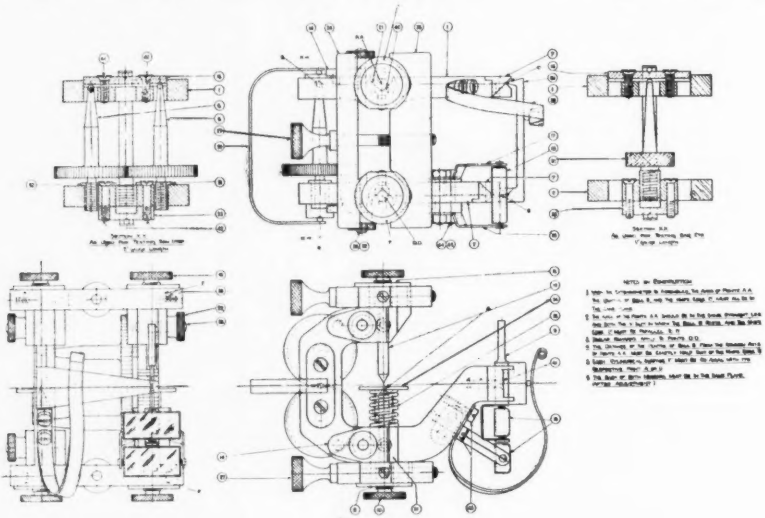


FIG. 4.

and the stresses occurring in valve springs. A further aid is a knowledge of the relation between the safe range of stress and the mean stress of the fatigue cycle, both values being expressed as fractions of the static ultimate strength of the material.

Goodman and Gerber have each suggested such a relation and both relations* can be expressed by the formula:—

$$R = R_r \{ 1 - (M/f)^x \}$$

where R is the limiting range when M is the corresponding value of the mean stress.

R_r is the limiting range when $M = O$.

f is the static ultimate strength of the material.

and x has the value 1 or 2 for the Goodman and Gerber laws respectively.

* In this case the law as originally enunciated by Goodman has been modified, the experimentally determined value of R_r being substituted for an assumed value of $R_r = 2/3\ f$.

The application of these laws is illustrated in Fig. 5.

Some experimental results agree with one of these relations, others conform to neither. For cycles of direct stress, the linear relation is generally a safe rule between the limits of $M=O$ and $M=\frac{1}{2}R$, *i.e.*, for alternating, pulsating and intermediate stresses. Recent experiments employing torsional stresses have shown that for some materials the value of R is not altered appreciably by wide variations in the values of M, but this lack of similarity to the laws governing corresponding direct stresses is presumably more apparent than real as the values of the stresses are calculated assuming elasticity and no allowance is made for the stiffening effect of the underneath layers.

The best known standard fatigue test for materials is that in which a turned specimen is loaded as a rotating cantilever and is subjected to alternating bending stresses. Figure 6 illustrates the type of machine used for this test at the

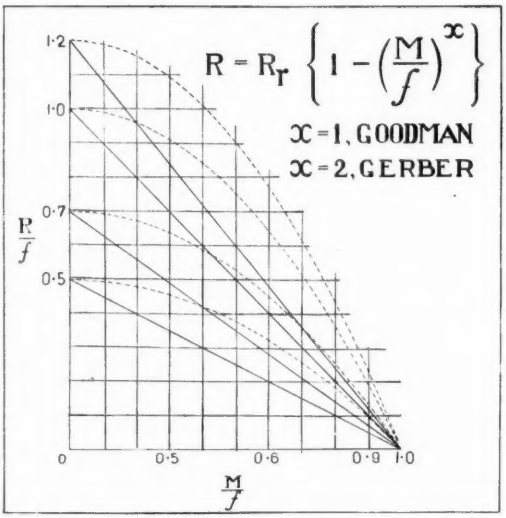


FIG. 5.

R.A.E. Its design is based on that of the corresponding type of machine at the N.P.L. Special auxiliary apparatus has been designed for representation of seaboard conditions of marine aircraft. Test pieces of (say) aluminium alloys may be enclosed in a chamber and subjected to the action of a 3 per cent. salt water solution spray, the temperature of the laboratory being thermo-statically controlled. The spray is directed on to the most highly stressed part of the test piece with sufficient force to penetrate the boundary layer. Non-stop tests are essential where corroding media are employed as the corrosion cannot be effectively stopped when once started.

Another standard type of fatigue test subjects a considerable length of the test piece to alternating uniform bending. This test is useful for testing the fatigue resistance of, for example, butt welded joints of tubes, etc., where localised wasting of the test piece would not be representative of service conditions.

Petrol pipes may be tested in this fashion or by reversed uniplanar cantilever bending.

It is not always convenient to apply the loads by dead weight as in the first example, and the required stresses may be induced in the test piece by inertial loading, spring loading or by treating the test piece itself as a spring and submitting it to a repeated cycle of strains, the load-strain relationship having been determined by prior static calibration. In the Haigh electrical machine a sinusoidal wave of simple tension and compression may be impressed on the test piece, the mean stress being controlled by spring loading. The Stromeyer is an example of alternating torsional fatigue testing machine incorporating inertial loading.

For a complete treatise on the fatigue of metals, including all the recognised methods of test, I will refer you to Dr. H. J. Gough's book on that subject and

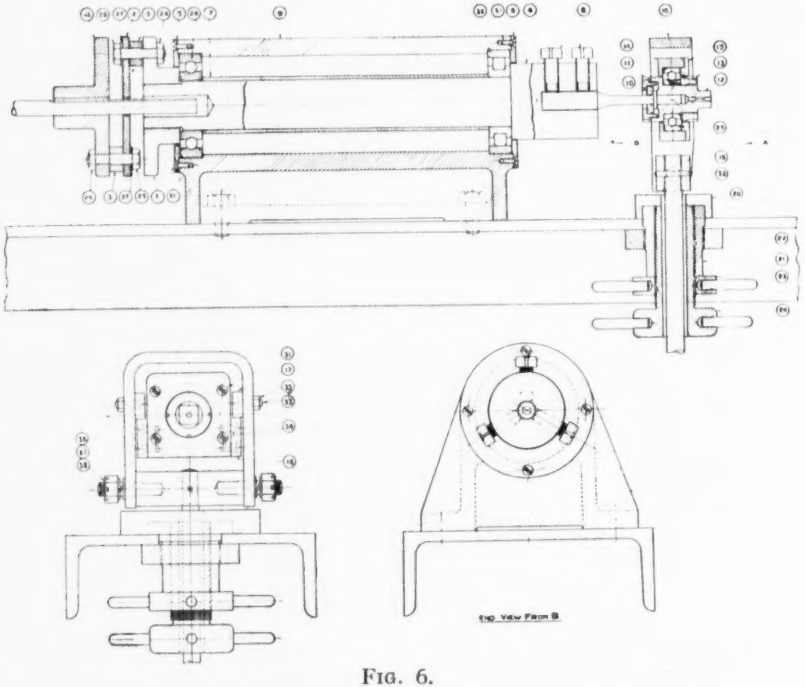


FIG. 6.

will conclude this section of my lecture by reference to a method of testing valve spring steel wires in torsional fatigue developed at the R.A.E. In this case a straight piece of wire about six inches long is gripped at one end by a stationary chuck of the spring collet type. The other end is similarly gripped by a rotatable chuck which impresses upon the wire a fluctuating twist of range $\frac{1}{4}$ maximum to maximum. The resulting mean stress and range of stresses are such as may be considered to be representative of aircraft valve spring service conditions.

As it is not possible to detect by visual inspection when failure first commences an indirect method of detection has been devised. Small concave mirrors are mounted some distance apart on the strained part of the test piece and images from a point-o-lite lamp are reflected on to a scale. These images, due to persistence of vision, appear as bands of light and are recorded by suitable

markers on the scale. There is no visible change in either the length or the position on the scale of these bands of light whilst the test piece responds elastically to the impressed stress system. As soon as, however, this condition is upset by the initiation of a crack, a progressive change in the length and position of either one or both of the bands (dependent on the position of the crack) is quickly evident. A slight yield by twisting in the direction of applied torque is also in some cases made evident by change in the position of both bands accompanied by a noticeable slight reduction in their lengths.

Bend Tests.—For metals where ductility is an essential, static bend tests may be used in addition to or as substitutes for elongation measurements in tensile tests. They are specially suitable for sheets and wires, as many metals in these forms give practically no elongation in tension even where the material is similar to that giving a large elongation in bar form. The edges of bend test pieces should be smooth and rounded off to avoid the premature initiation of cracks due to concentration of stress at sharp corners or ragged edges.

Reverse Bend Tests are suitable for round wires, streamline wires and swaged rods, and have the advantage over simple bend tests that they give numerical results which can be compared, instead of being merely a test which the sample passes or fails to pass.

Torsion Tests are used as a standard method of measuring shear strength. They are also used for detecting brittle wires among the very hard drawn strands of a wire rope. Another brittleness test for wire is the wrapping test where the wire is to be wrapped eight times round its own diameter and then unwound again (except the last turn) and must show no fracture.

Indentation Hardness Tests give a useful indication of the ultimate strength of parts where the relationship between hardness number and ultimate strength is known. They are also, with this limitation, an invaluable process inspectional test to check the effectiveness of any given heat-treatment or cold work process used in manufacture.

The Notched Bar Test (Izod) is principally useful as an indication as to whether the heat-treatment which has been given to a steel is satisfactory or not. Its numerical value is dependent on the class of steel, and it is, therefore, impossible to compare two steels solely upon the basis of the absolute Izod value obtained from them. In general, however, experience has shown that the existence of a good Izod value is certainly of advantage in an engine part.

Part II. Tests of Components

Given a knowledge of the strength of materials, it is equally essential to know how best to use these materials in the construction of light weight aircraft components of sufficient strength and stiffness. The strength of but few components is amenable to calculation with sufficient accuracy to obviate the necessity of checking by the mechanical testing of samples. Small parts such as bolts, shackles, etc., detail components such as spars, struts, ribs, landing wheels, etc., and major components such as wings, hulls, fuselages, etc., are all tested to ensure that their design fulfils the desired strength requirements and to provide design data for future use.

Tests of Spars.—Several hundreds of spars have been tested at the R.A.E. alone, and in many cases contractors have submitted for test a number of alternative designs of spar for a given aircraft before deciding which to adopt.

The type of test chosen is dependent on the anticipated service conditions of loading.

For *biplane spars* in order to assign the allowable maximum stresses (including the load factor for the appropriate aircraft), a convenient practice is to test at least two lengths of spar under combined end compression and bending. The ratio of end compression to bending in the two cases is varied so that for

one spar the case of estimated maximum end load combined with the corresponding bending moment is represented and for the other spar the case of maximum bending moment combined with the corresponding end load is represented. From the results of these two tests, the stresses for any intermediate combination of end load and bending moment may be estimated on the usual assumptions.

In general, end compression is applied in a horizontal testing machine to a spar, of length equal to that between two points of contraflexure in the actual spar represented, with the major axis of cross section vertical and with horizontal pin joints at each end (the longitudinal axes of the pins usually lying in the plane of the minor axes of the spar cross section). Bending moments are applied by vertical downward shear loading at two symmetrically placed loading boxes built into the spar about one-quarter of the length from each end. Horizontal lateral and anti-twist support is provided to represent the effect of adjacent parts of the wing structure.

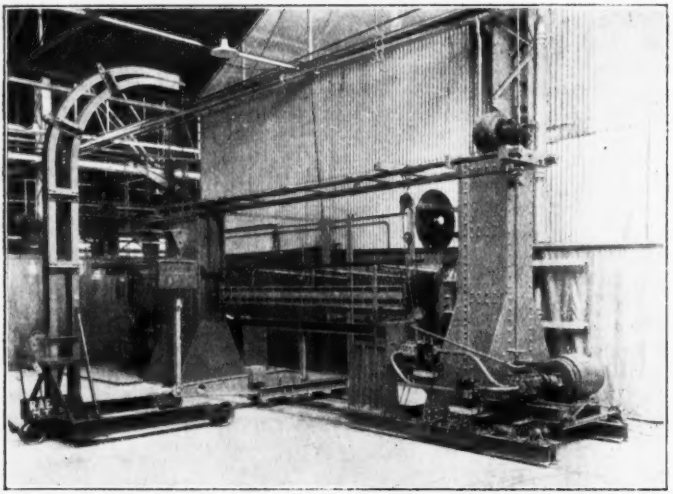


FIG. 7.

The end compression and lateral shear loads are applied incrementally in such a manner that their ratio is brought to the agreed value at the completion of each increment, when the vertical deflection of the spar at mid-length is measured.

From the test readings, the apparent total ultimate stress developed at the upper skin at mid-length of the spar is estimated as the sum of

(*a*) the direct compression stress (end load/area);

(*b*) the bending stress due to lateral shear loads (vertical lateral shear load reaction × distance between loading box and end pin joint/modulus of section);

and (*c*) the bending stress due to the product of end load and deflection of the spar (end load × deflection at centre relative to ends/modulus of section).

The stresses at other points in the central cross section of the spar are similarly estimated assuming elastic flexure.

A test of this kind is shown in Fig. 7, which also illustrates one form of spar testing apparatus developed at the R.A.E. This testing machine, a general

arrangement of which is shown in Fig. 8, will accommodate test pieces up to 100ft. long with easy access at all points along its length. End compression loads up to 100 tons may be applied and provision is made for lateral loads up to 10 tons.

Fig. 9 shows the test of a spar with a single lateral load. As this test spar was too long for the use of a bridge deflectometer, deflections were

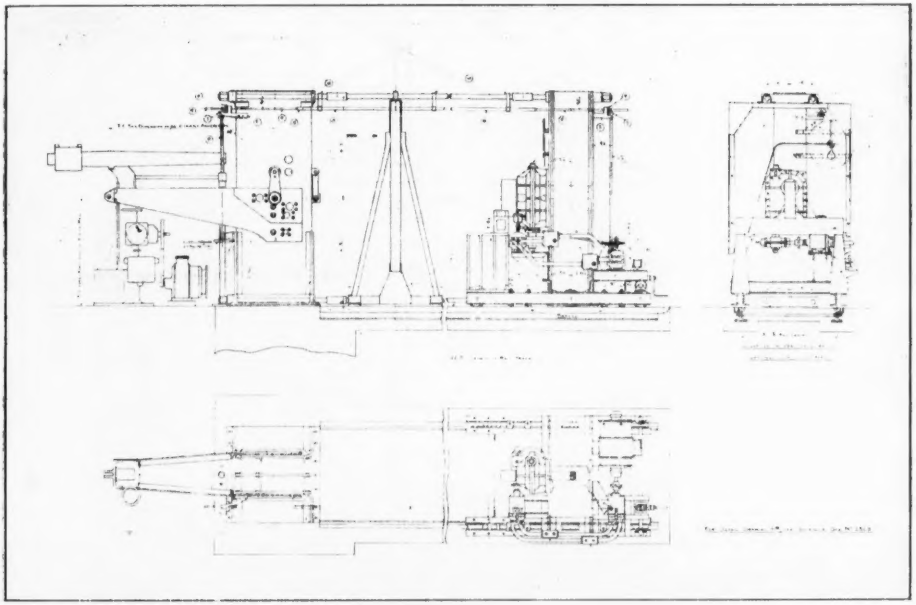


FIG. 8.

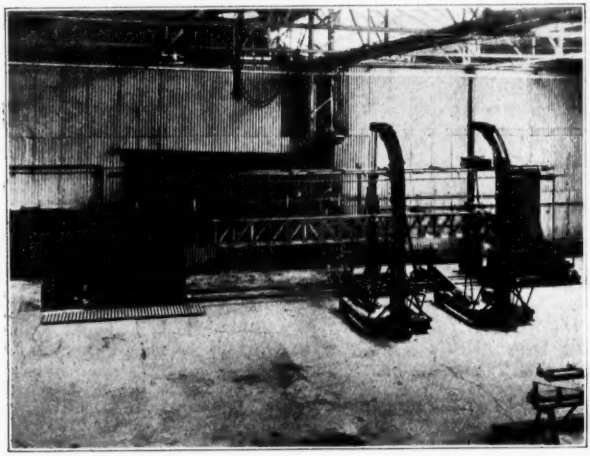


FIG. 9.

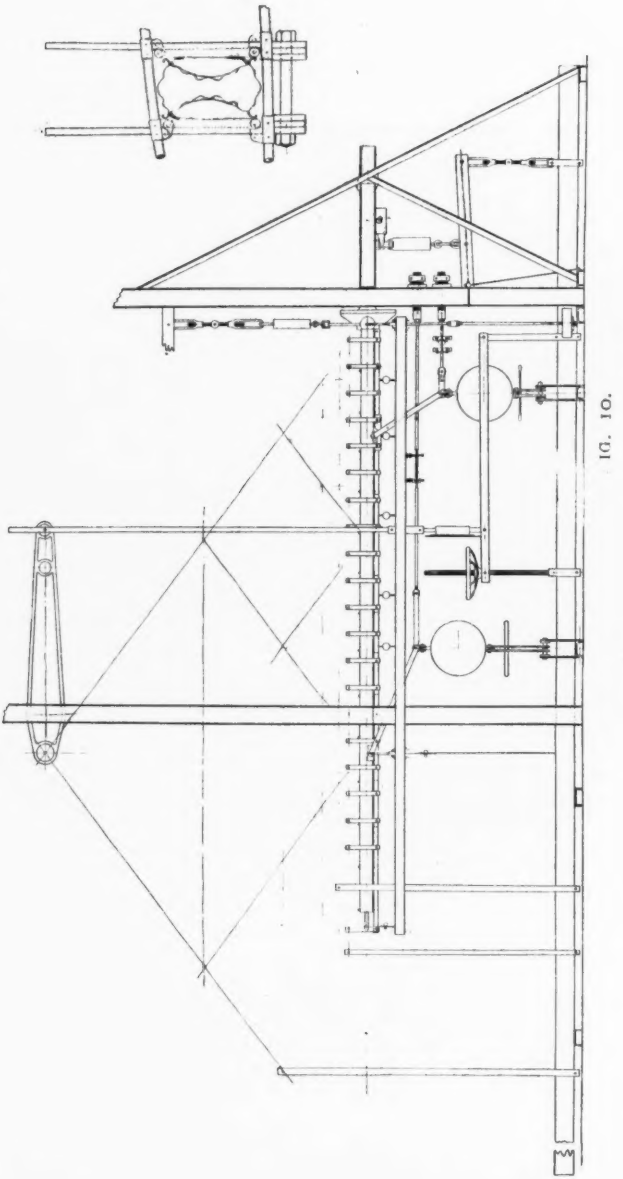


FIG. 10.

measured by means of a dumpy level and vertical scales attached to appropriate points of the spar.

As the spar of a biplane is a continuous beam, the bending moments, etc., in the overhang and in each of the bays have a mutual interaction.

Berry and others have evolved methods of estimating the bending moments and deflections in such parts. Where it is necessary to check these calculations, a test of the type shown in Fig. 10 is made. The spar is mounted in the normal attitude and the lever distributor applies the necessary lift forces, inclined ties being provided to represent the resultant forces from struts and lift wires at

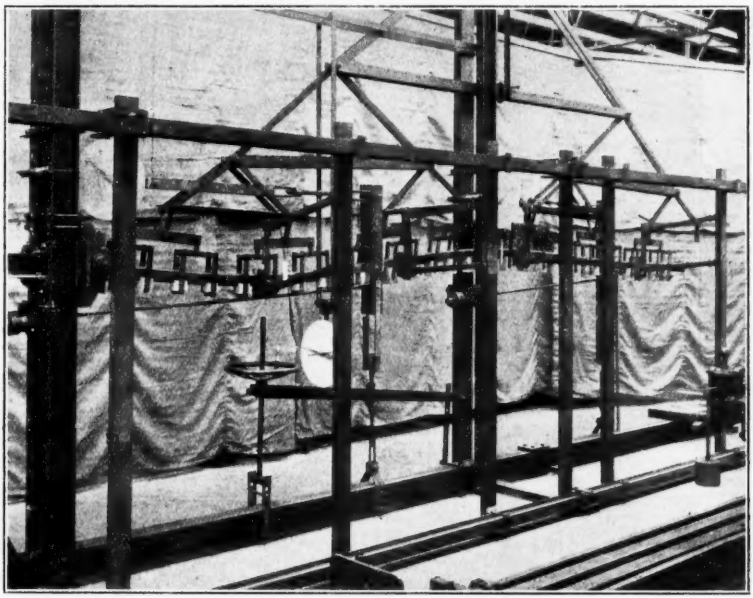


FIG. 11.

appropriate points on the spar. The root of the spar is housed in a socket fitted with a horizontal pin joint and, where necessary, this pin joint is eccentric to provide an end fixation moment. The convention of allowing no relative vertical movement of the root pin joint and the points of strut and lift wire attachment is observed as in the Berry analysis. Provision is made for measuring the total air load applied, the vertical and horizontal reactions at the root of the spar and at the points of strut and lift wire attachment. Deflections of the spar at various points are also measured. Parallel motion links mounted on the test frame prevent premature failure by horizontal lateral deflection or by twist. The final stage of a test of this kind is shown in Fig. 11, where it will be seen that the inner bay is deflected downwards, against the air load, due to the overwhelming effect of the bending moment transmitted from the long outer bay. This condition was predicted by prior calculations.

For *monoplane spars* either a cantilever or a semi-cantilever test is made on a sample length of spar as the case represented requires.

Fig. 12 shows a method of testing the overhang of a spar as a cantilever. Provision is made for the attachment of the root of the spar to the vertical limb of the test frame so that the spar lies in the normal attitude above the horizontal limb of the test frame and parallel motion links prevent undue lateral deflection or twist of the spar.

In this case a single upward load is applied at the tip of the spar by means

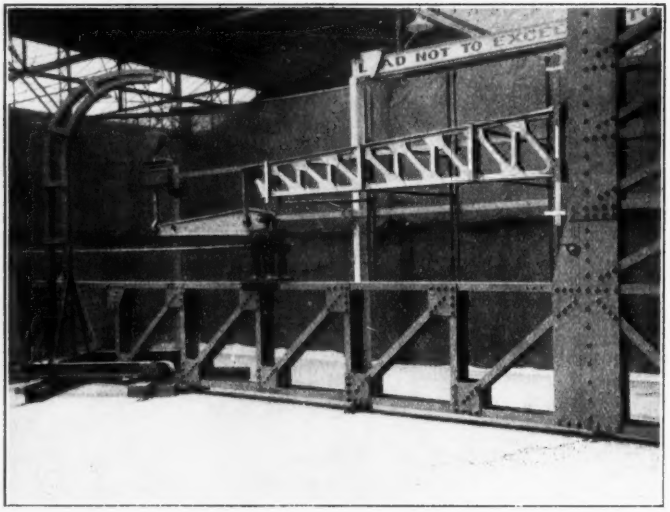


FIG. 12.

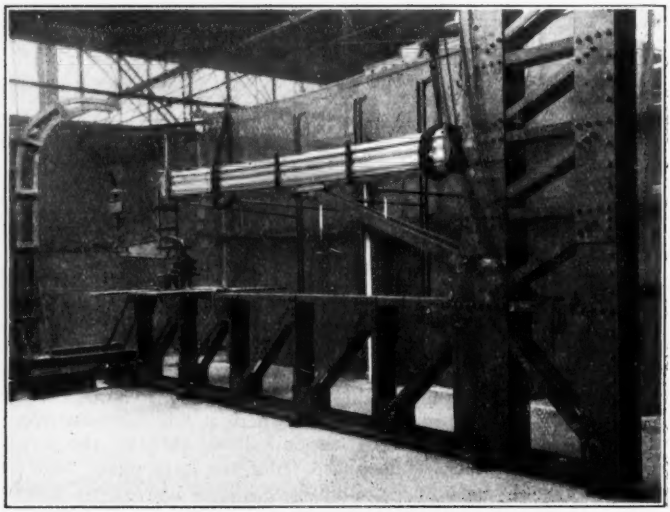


FIG. 13.

of the roller bearing lever at the left, one end of the lever being raised by screw jacks with distant control and the other end by a crane with a suspended weigher in series for measuring the load applied. In some cases a number of vertical upward loads are applied at different points along the spar. Deflections of the spar are measured by a level and vertical scales mounted on the spar. Rotation at the root is measured by an alidade and two horizontal scales mounted on a vertical arm attached to the spar near the root.

Fig. 13 shows a similar test, but with semi-cantilever loading.

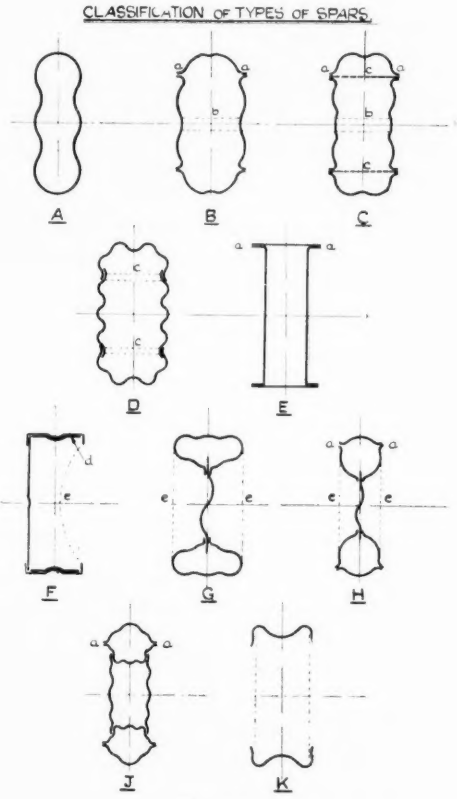


FIG. 14.

Classification of types of spars tested.—The various sections of spars tested may be classified as in Fig. 14. All sections are shown with the major axis of cross section vertical.

Section A represents spars made from solid drawn tubes of sections which may be circular ended with flat or horizontally corrugated sides. Spars of these sections, though not much stronger for a given weight than spars of the equivalent plain circular sections, are appreciably stiffer against bending in the plane containing the longitudinal axis of the spar and the major axis of cross section.

Section B represents spars of longitudinally corrugated tube substantially oval in section, built-up of two flange members and two web members connected

together by rivets or their equivalent at "lips" "a-a-a-a." These lips are placed as near as may be convenient to the minor axis of cross section to minimise their stress due to bending. The flange and web members may each comprise one or more longitudinally corrugated strips (multiple strips usually nesting together). The webs are in some cases connected by tubular or other type distance pieces "b."

Section C is somewhat similar to Section B, but has, in addition, flange ties or stiffeners "c" of flat strip perforated by lightening holes.

Section D is similar to Section C, but has tubular flange stiffeners.

Section E is substantially rectangular in section with lips at each corner. The flanges may be either of extruded section or of one or more longitudinally corrugated or flat strips, in some cases combined with angles. The webs may be of one or two transversely or longitudinally corrugated strips.

Section F may be formed from one or more longitudinal strips, sometimes with additional strips "d" added to reinforce the flanges. The web member of this section is usually reinforced by vertical stiffeners "e" fitted at some definite minimum pitch along the length of the spar.

Section G comprises two tubular flanges of longitudinally corrugated strips connected to a single web member which may be longitudinally corrugated or flat, with or without occasional vertical stiffeners "e." The rivets in this section are as close to the minor axis of cross section as the particular shape of flange used will allow.

Section H is similar to Section G, but in this case the flanges are made up of three strips (one flange and two cornice strips). There are, therefore, "lips" in this section somewhat similar to those in Sections B and C. Occasional vertical stiffeners "e" are sometimes added, as in Section G.

Section J is like Section H, but has two longitudinally corrugated web strips.

Section K takes the form of a braced girder (usually lattice or warren). In some cases flat external web plates perforated by lightening holes are added.

Classification of Type of Failure Development.—The initial and subsequent stages of failure of any given spar vary, of course, according to the spar construction and conditions of loading.

Considering first a test spar of Section G, loaded as a beam with longitudinal compression and lateral flexure as previously described; as the loads are gradually increased the whole spar develops a simple bow (concave upwards) in a vertical plane. This vertical bowing may be accompanied by bowing (chiefly of the upper boom which is in maximum compression) in a horizontal plane and dependent chiefly on (1) what horizontal lateral supports are provided in the test, and (2) the end fixation moments provided by the horizontal loading pins at each end of the spar.

Further increase of the loads may (and generally does) produce local waving of the upper flange member and/or web member(s). The loads at which this waving occurs are dependent on (1) the ratio of the effective radius of transverse curvature of the strip to its thickness at various points in the spar section, (2) on the physical properties of the materials of the various strips, and (3) the effective support of local stiffeners, if any.

At the ultimate loads borne by the spar, these waves usually develop into a local buckle involving both the flange and web, but chiefly concentrated at one side of the boom according to the proportion of horizontal to vertical bowing present during earlier stages of the test. Relative rigidity of a section about a vertical axis of the cross section and/or ample lateral support of the upper boom, may, however, tend(s) to constrain the failure of the section to take place with horizontal symmetry.

It may thus be seen that the chief problem before the designer of this type of spar (*G*) is to produce a tubular boom which can fulfil the functions of a strut under end compression combined with lateral loads and restraints.

Considering the top boom, which is in maximum compression, the lateral restraint, provided by the spar itself, is supplied by the web and the bottom boom, and is chiefly effective in a vertical plane, whereas lateral restraint in a horizontal plane is chiefly provided by other members of the wing structure. Torsional stiffness of the spar and of the remainder of the wing structure would account for additional restraints from these respective sources in the inverse order, *i.e.*, horizontal restraint from the bottom boom and web, and vertical restraint from the other members of the wing structure. These additional restraints are, however, usually small compared with those first described.

Considering now sections of all the types described above, the possible types of failure involving the upper boom of types *G* to *K* or the upper flange or corresponding portion of types *A* to *F*, may be classified as follows:—

(*a*) Failure by bowing as a complete strut.

Initial bowing may be elastic and subsequent bowing may, or may not, result in permanent deformation before the crippling load is reached.

(*b*) Failure by localised waving of the walls.

Here also the initial waving may be elastic, but may later be carried so far as to give permanent set.

(*c*) Failure by local longitudinal contraction of the wall unaccompanied by appreciable waving.

This type of failure is by direct compression of the material, and is chiefly dependent on the physical properties of the latter.

These three contributory types of failure are present in various proportions in all spar failures of the kind at present under consideration.

Failure of type (*a*) is chiefly noticeable in the test either of a long slender spar, where bowing of the whole spar occurs in a vertical plane, or of a deep narrow sectioned spar with not too much lateral support, where horizontal bowing of the upper flange or boom occurs in addition to the vertical bowing of the whole spar.

Failure of type (*b*) is prominent in the majority of tests here considered, and is dealt with more fully later.

Prominence of failure of type (*c*) is uncommon, being confined to spars with thick walls and material whose yield stress is not very high, *e.g.*, type *A* with thick walls made of duralumin.

Although the average loading conditions, *i.e.*, the usual ratios of direct compression stress to stresses due to bending, produce in a large majority of sections initial failure in the upper boom or flange, the stress distribution in the section combined with the actual properties of the section may be such that failure appears to develop first in the web member. This form of failure may be expected to occur where the direct compression stresses are large compared with those due to bending, and/or the local stability of the web members in longitudinal compression is less than that of the flange members.

Intermediate cases arise where it is not easy to decide whether the flange or the web is the seat of initial failure.

Apart from flange or web failures of the types described above, failure occasionally occurs by shear distortion of the web members. This type of failure is confined to the webs between the loading boxes and the ends of the spars, and usually occurs only when the lateral shear loads are large compared with the end loads. Like all other types of failure, it is, of course, dependent on the form of the section, and is influenced by the presence of lightening holes, corrugations, stiffeners, etc.

Classification of Types of Local Waving relative to particular types of spar section. Section A.—For pure end compression, waving is initiated in the sides, *i.e.*, in that portion of the section corresponding to the web members of other sections.

The addition of bending moment, as described above, displaces the seat of initial waving towards the top of the section.

Sections B, C, D, G, H, and J.—For pure end compression, waving will originate in that part of the section which is least stable under end load. In most cases this is the web, as spars are usually designed to resist combined end compression and bending.

The addition of bending moment tends to shift the seat of initial waving towards the upper flange, the amount of displacement depending on the relative intensity of bending to end compression.

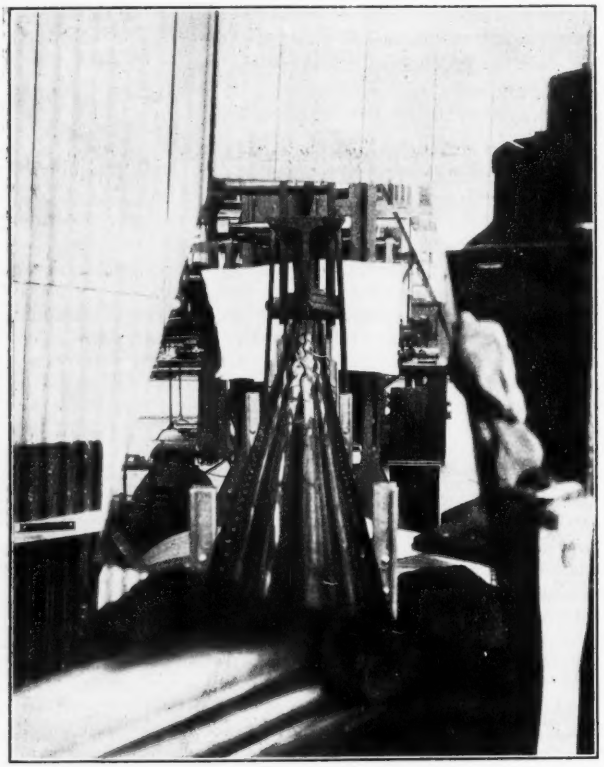


FIG. 15.

Apart from the influence of these regions of relatively low stability, there are, in many sections, other determining factors, such as the presence of free lips, as shown in Sections B, C, H, and J, at " a "; local weakening, due to the presence of lightening holes, rivet holes, etc.; and the presence of local stiffeners such as those shown dotted in Sections B, C and D, at " b " and " c."

Section E.—Where the web member of the section is transversely corrugated it may be incapable of taking appreciable end load, and in such sections waving will start at the horizontal lips " a-a-a-a."

Distortion of the web in some cases affects the flange which is in maximum compression and helps to initiate waving of the flange.

Section F.—Spars of this section are usually divided into panels by transverse stiffeners fitted at some definite pitch along the length of the spar. Waving is, therefore, to a large extent controlled by the position of these stiffeners and may occur first either in the web or flange, according to their relative stresses and stability under longitudinal compression.

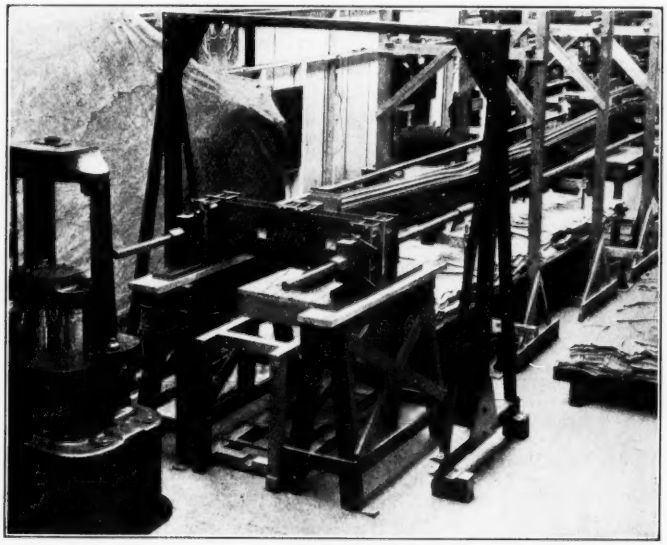


FIG. 16.

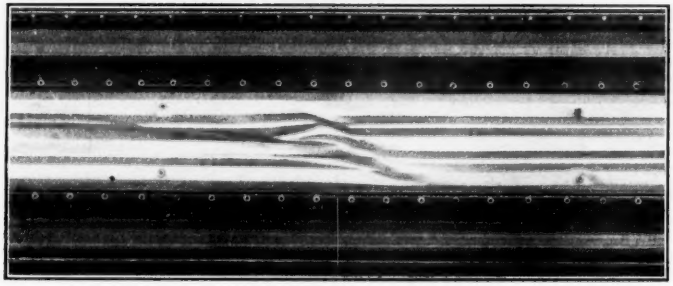


FIG. 17.

Section K.—Spars of this section, like those of Section F, are usually divided into a number of panels along their length. In many cases the web is not continuous and therefore does not take a proportionate share of end compression. The upper flange in this case may be regarded as a continuous beam, the pitch of waving being usually a simple multiple of the pitch of the panel points.

Fig. 15 shows elastic waving of the flange of a spar under test, and Fig. 16 shows similar waving developed to collapse of the flange. Incidentally, this figure is also interesting as it shows the immediate forerunner of the R.A.E. 100-ton

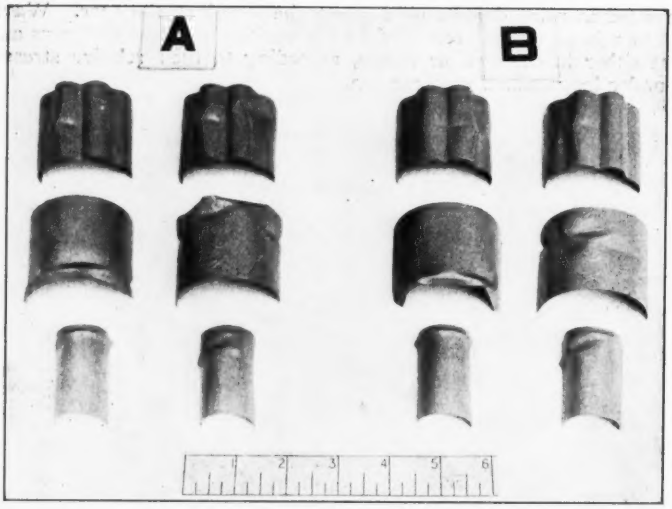


FIG. 18.

FIG. 19.

horizontal testing machine, and an early application of the hydraulic rams subsequently used for wing tests, as will be described later.

Fig. 17 shows an unusual type of failure, due to combined shear and transverse compression in the web. It may be combated by suitably-designed rib posts or other forms of vertical stiffeners.

The prevalence of local crippling of the constituent strips under end compression would lead to the suggestion that the criteria of thin strip material for the manufacture of aircraft spars, etc., might be found from compression tests of short lengths of corrugated strip, the ratio of radius of transverse curvature to wall thickness being appropriate to the design of section for which the material is intended and to the physical properties of the material itself. Test pieces of this kind are shown in Fig. 18 and convenient test apparatus in Fig. 19.

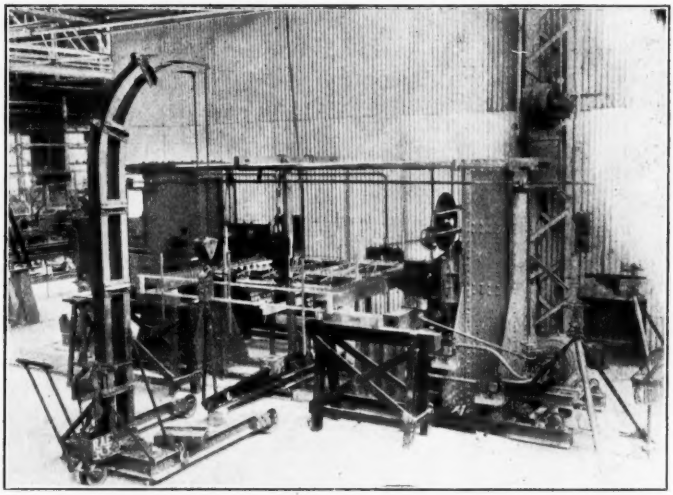


FIG. 20.

Tests of Main Planes.—The strength of some spars is so dependent upon other portions of the main plane structure, such as adjacent spars, ribs, metal covering, etc., that tests of a portion of the plane are advisable.

Fig. 20 shows a test of a portion of a two-spar biplane wing, complete with two compression ribs, etc., and four former ribs.

In this case the spars were staggered and information was required on the effect of this on the former rib failing loads. The end load was suitably apportioned between the front and rear spars by means of a lever system, while the air load was applied by means of a distributing link and lever system similar to that used in rib testing. The horizontal and vertical deflections and twists of both spars were measured by levels, alidades and scales arranged as shown in the figure.

Figure 21 shows a method of measuring the lateral stabilising forces required to keep within small limits the lateral deflection and twist of a spar under given loading conditions.

Up to the present time, the strengths of most main planes have been determined by the application of dead weight loading to the planes when attached to a fuselage or other suitable framework, the deflections being measured

by scales and optical levels. This method, which has been developed by Messrs. W. D. Douglas and A. W. Clegg at the R.A.E., was described by them in R. and M. 476. Similar methods are also used for the determination of the torsional and flexural stiffness of main planes, etc.

The continued increase in size and strength of aircraft components, has, however, created cases outside the range of practical application of this method, and alternative methods of test have had to be devised.

A case in point is the multi-spar wing, designed and in process of development by Mr. H. J. Pollard and the Bristol Aeroplane Company.

As it was anticipated that a large number of tests would have to be made on the same wing structure, suitably modified between successive tests, the method chosen had to be quick in operation and such that the damage due to failure

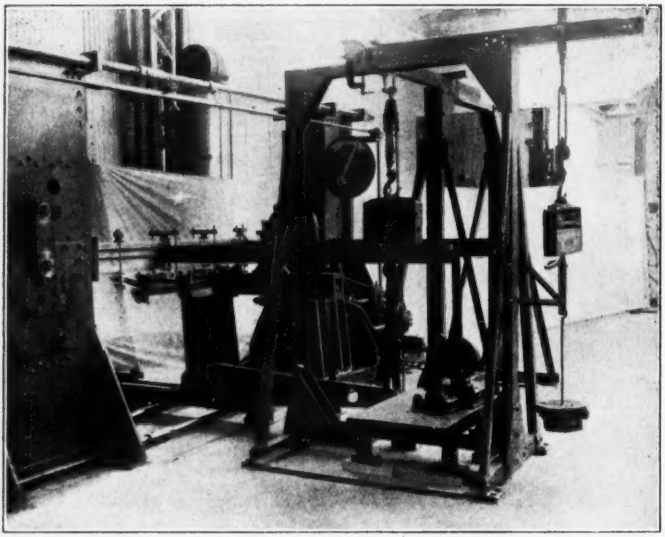


FIG. 21.

would be confined to but a small part of the structure. The size and strength of the plane called for the application of large loads. These requirements were met by the method illustrated in Figs. 22 and 23. It will be seen that this is an elaboration of the test of the single cantilever spar. Instead of a single spar the test structure consists of seven spars interconnected by ribs and bracing, and, instead of a single concentrated load at the spar tip, each spar is loaded at 20 separate points.

So far the loads have been graded from spar to spar and along each spar to represent (*a*) normal flight conditions, or (*b*) flight conditions with aileron in action, but provision is made for the representation of other service conditions of loading. Levers are used for chordal distribution of load, and hydraulic rams (of a pattern previously developed at the R.A.E. for general testing) each in series with a screw jack, are used for longitudinal distribution of load. The total load applied is measured by the hydraulic pressures in the rams as indicated by Bourdon type pressure gauges and dead-weighted rams of relatively small diameter. The effect of friction of the rams is minimised by oscillatory rotation of the rams during application of load increments.

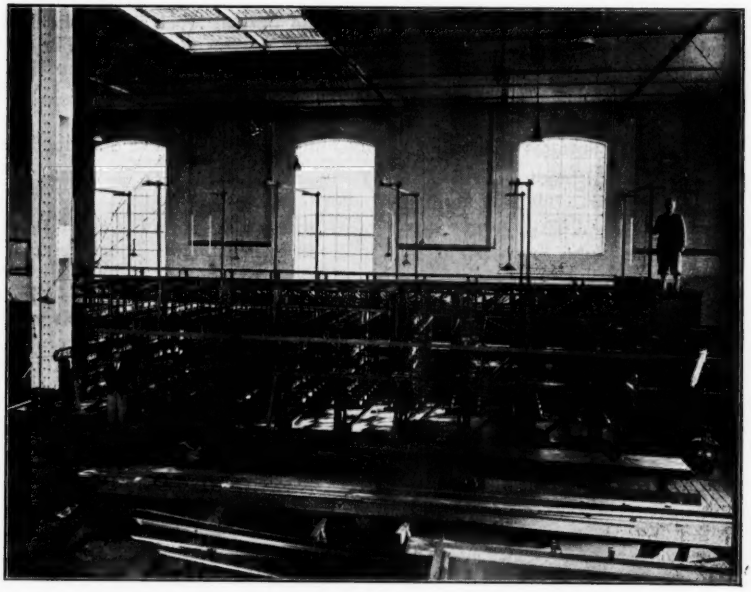


FIG. 22.

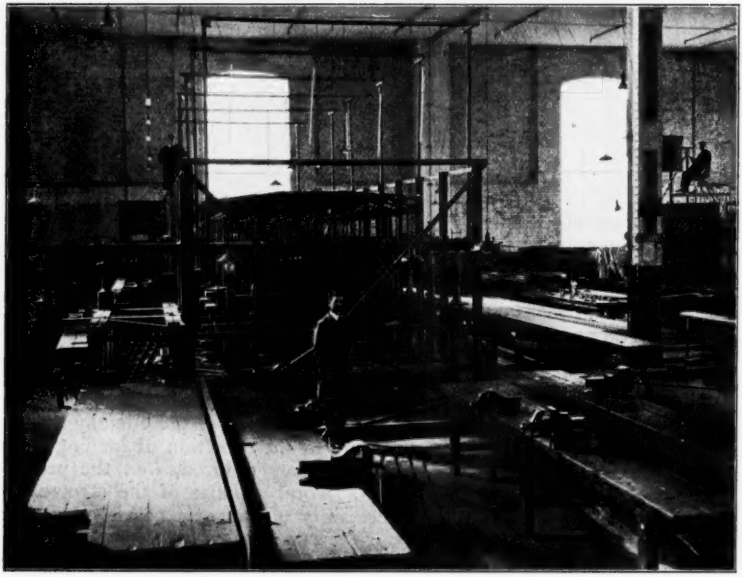


FIG. 23.

Deflections of the wing are measured by an optical level and vertical scales attached to various points on the upper surface of the structure. The results obtained from this method of testing the wing structure have fully justified the initial cost of the apparatus, which is, of course, high compared with that required for dead-weight loading methods.

The chief advantages of loading by machine methods as here described are :—

(1) Large loads may be quickly and safely applied with the assistance of relatively few labourers.

(2) All the exterior of the structure under test is visible throughout the test and may be closely inspected with safety. It is frequently of especial importance to be able to examine continuously during certain stages of a test those portions of a test structure which are in compression, *e.g.*, wings, which may be tested in any attitude, are preferably tested so that the surface in maximum compression is on top where it may easily and safely be seen (from a superstructure if necessary) under favourable lighting conditions and whilst the loading is being applied.

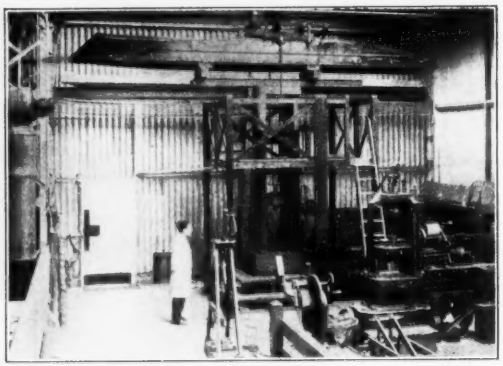


FIG. 24.

(3) If at any time during the test it is desired to observe how much distortion is permanent, or for any other purpose, the load may be quickly released.

(4) The loading apparatus does not appreciably interfere with the distortion measuring apparatus.

(5) The application of load may be arrested as soon as the first signs of collapse are visible, and, if necessary, the load may be quickly released and local stiffening or reinforcement applied to the test structure before proceeding with the tests.

(6) As a stable system of loading is employed, if there is any appreciable distortion of the test structure in the direction of loading, the load is automatically and instantaneously reduced, *i.e.*, the progression of failure is strictly limited. Total collapse of the structure under test, with consequent uncertainty of cause of collapse, is therefore impossible.

(7) On completion of the test the residual load may be quickly released, and the work of repair or replacement of the test structure may proceed with a minimum delay.

Tests of Floats, Hulls and Monocoque Fuselages.—Up to the present time there has been a scarcity of mechanical test results to support the allowable ultimate skin stresses usually assumed in stressing aircraft floats, hulls and monocoque fuselages. Design data is, however, now being accumulated from tests of structures of this type. Fig. 24 shows the test of a duralumin float, the

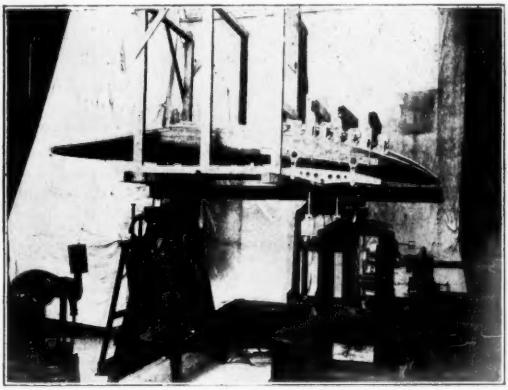


FIG. 25.

loading conditions representing normal taxying. In Fig. 25 a timber float is shown tested under conditions representing alighting on the water. Comparative tests of timber and duralumin floats show that metal floats are both lighter and stronger than timber floats.

Fig. 26 shows diagrammatically a method of testing a large metal hull. Case (1) represents down load on the tail during flight, and case (2) represents landing on two waves respectively near each end of the waterline.

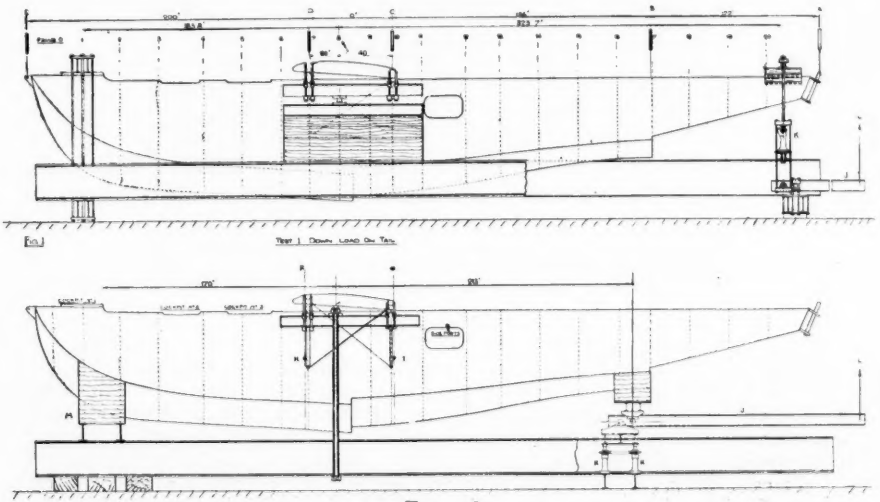


FIG. 26.

In both cases it may be seen that the hull was in the normal attitude, and loads were applied by a system of beams, levers, compression blocks and ties, the reactions being taken by two stout beams placed one at each side of the hull.

In case (1) a downward load on the tail portion of the hull was applied by

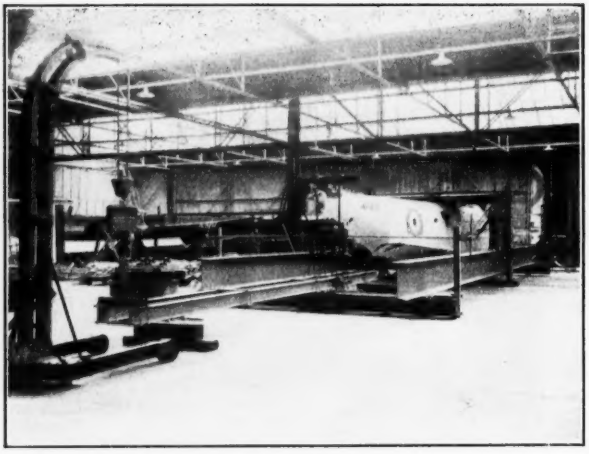


FIG. 27.

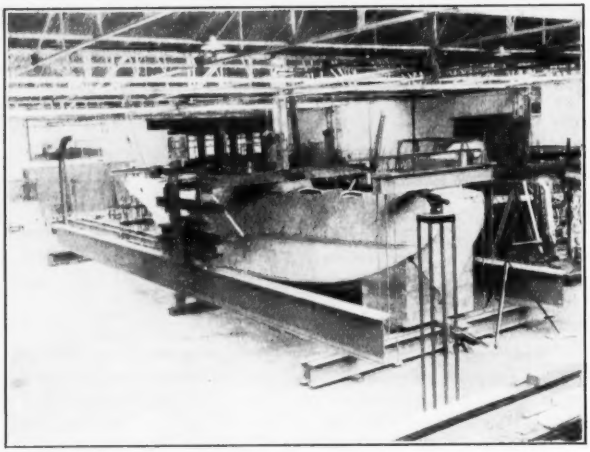


FIG. 28.

means of a lever (J) and a tension straining member (K), the load applied being indicated by a suspended weigher (L). Reactions on the hull were provided by upward vertical support of the centre section of the lower wing and a downward thrust on the fore body of the hull, the hull being internally reinforced to prevent local crushing at this point.

In case (2) an upward load was applied near the rear step of the hull by means of a lever (J) and four screw jacks (KK), the load being indicated by the

suspended weigher (L). A downward vertical reaction was applied at the centre section of the lower wing, and a vertical upward reaction at the fore body of the hull by a shaped block (M), the internal reinforcement of the fore body remaining in position to function as in the first test.

General views of case (1) are shown in Figs. 27 and 28, and those of case (2) in Figs. 29 and 30.

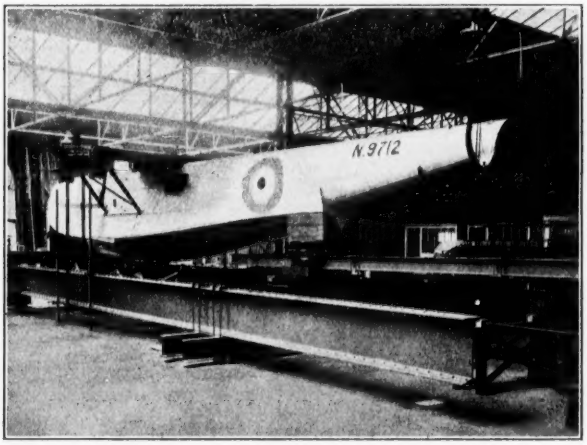


FIG. 29.

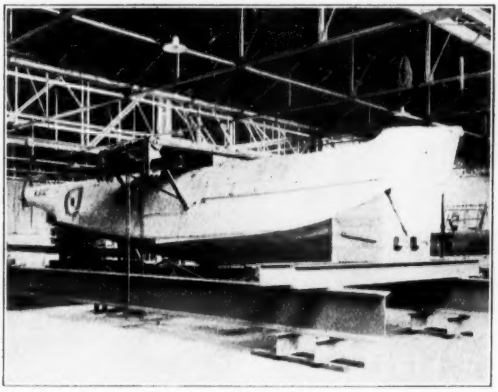


FIG. 30.

In both cases the distortions of the hull in a vertical longitudinal plane were measured at each increment of load by means of an optical level, and scales attached to the upper surface of the hull. Measurements were also made at each increment of load of the contraction or dilation of the hull, both horizontally and vertically, in two transverse planes near the front and rear spars respectively.

Tests of complete hulls such as the foregoing are, of course, dependent on the availability of a hull which is no longer required for other services. For

preliminary tests of the design of hulls and monocoque fuselages, part sections of the structure are in some cases first made and tested before proceeding with the construction of the complete structure. Test pieces of this kind are shown in Fig. 31. AA is a portion of the side of a monocoque fuselage and was tested

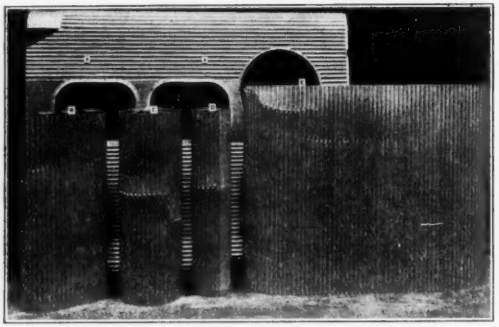


FIG. 31.

in shear. B, C, and D are cylindrical test pieces made to determine the effect of various degrees of mean curvature on the ultimate compressive stress developed. E represents the bottom of the same fuselage, and was tested in compression as shown in Fig. 32.

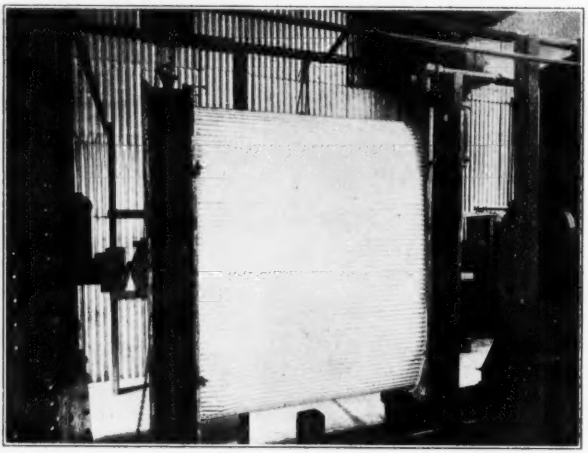


FIG. 32.

In order closely to approximate to uniform stress in this large section, very stiff end plates were necessary, those illustrated being of a deep and heavy girder section. The loading pins are the same as used for tests of large spars and have provision for initial alignment in a horizontal plane. Lateral support of the centre of the test section relative to its ends was provided.

Tests of Girders.—Fig. 33 shows a girder over forty-three feet long tested as an eccentrically loaded ball-ended strut. In this test, fourteen measurements of

the distortion of the girder and strains in twenty-four selected bracing wires were measured for each of sixteen increments of load in approximately two hours.

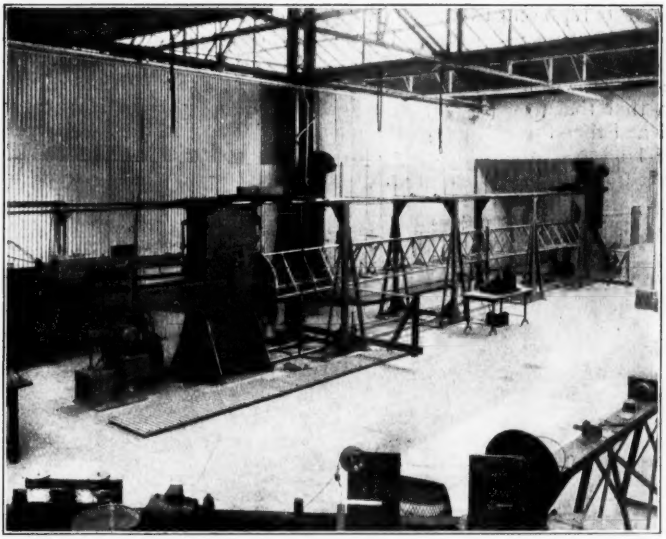


FIG. 33.

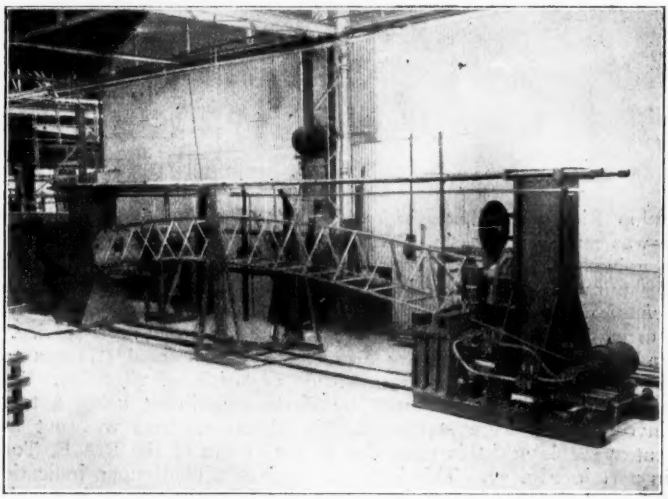


FIG. 34.

Fig. 34 shows the compression test of a cambered girder, and Fig. 35 a tension test of the same girder.

Tests of Landing Wheels.—Samples of all new types of landing wheels are tested to destruction to determine for what weight and type of aeroplane they are suitable.

The tests represent two conditions of loading: (1) Landing on two wheels with the undercarriage so distorted that the wheels are inclined top inwards at 15° to the vertical. This test is, where possible, carried to destruction. (2) Landing on one wheel which is inclined top outwards at 15° to the vertical and with drift so that the ground reaction is inclined in the opposite sense at 15° to the vertical and therefore at 30° to the plane of rotation of the wheel. This test is a proof load test and is usually only carried up to a load of about half that anticipated in the first test. Fig. 36 shows a test of the first type by Messrs.

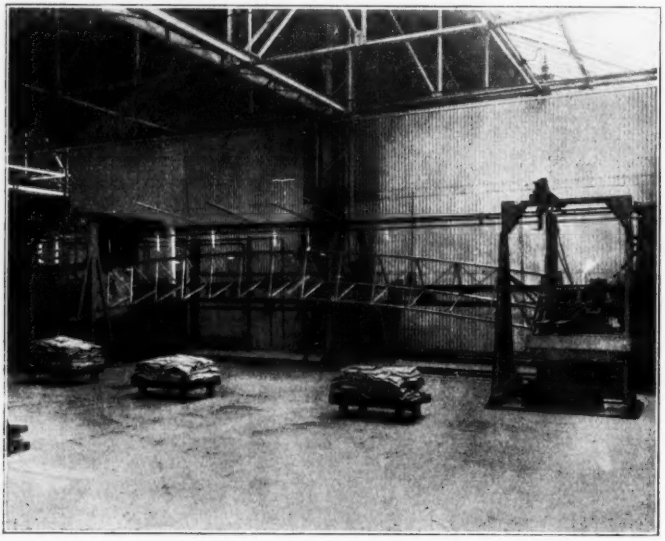


FIG. 35.

The Dunlop Rim and Wheel Co., who kindly lent me the slide. The testing machine was made by Messrs. Denison, of Leeds, to a design founded on experience gained at the R.A.E., where up to that time most wheels had been tested for purposes of airworthiness approval.

Mechanical Test of Engine Components.—The strength of nearly all new designs of engine parts is tested before the parts are put into service. Crankshafts and certain other parts are also tested for torsional stiffness to provide data for the calculation of critical speeds of torsional vibration. The latter determination may also be obtained by direct experiment using a torsiograph. When investigations were started in this direction, there was not a suitable instrument available and this gave rise to the design of the R.A.E. Torsiograph shown in part in Fig. 37. This instrument gives a continuous indication of the torque twist relationship in an airscrew drive whilst the engine is running. The record may take the form of a series of torque-twist polar diagrams for selected engine speeds and from these the speed at which maximum torsional oscillation occurs can readily be seen. This instrument has for some years been used by the Engine Research Department, R.A.E., where the original Mechanical Test Department design has been considerably improved in detail

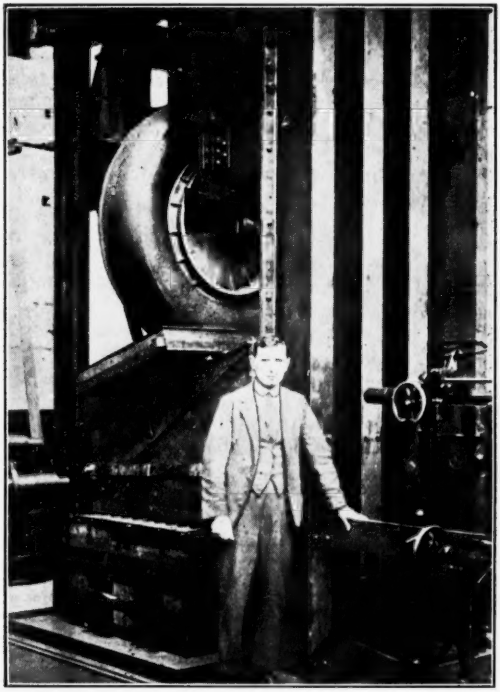
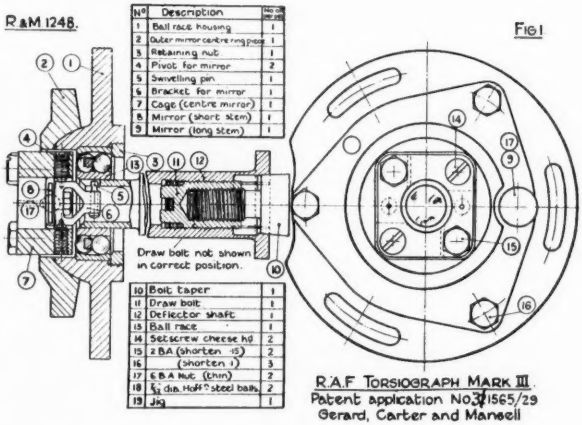


FIG. 36.



No	Description	No. off
1	Ball race housing	1
2	Outer mirror centre ring piece	1
3	Retaining nut	1
4	Pivot for mirror	2
5	Swivelling pin	1
6	Bracket for mirror	1
7	Cage (centre mirror)	1
8	Mirror (short stem)	1
9	Mirror (long stem)	1
10	Bolt taper	1
11	Draw bolt	1
12	Deflector shaft	1
13	Ball race	1
14	Setscrew cheese hd	2
15	2 BA (shorten 15)	2
16	(shorten 1)	3
17	6 B.A. Nut (thin)	2
18	5/32 dia. Hoffs steel balls	2
19	Jig	1

FIG. 37.

Conclusion

I hope that my lecture has illustrated the primary importance of mechanical testing in aircraft construction not only at establishments such as the R.A.E., but also at all aircraft constructors' works. I hope I have also made it clear that this important branch of aircraft design is not of the type which it is expedient to entrust to those with no prior design experience, but calls for sound theoretical training coupled with a long and close acquaintance with the best constructional practice.

I have to add that only my own personal opinions are expressed or implied in the lecture.

THE SPINNING OF AEROPLANES

BY

S. SCOTT-HALL, M.Sc., F.R.Ae.S.

(*Paper read before the Yeovil Branch, February 12th, 1931.*)

Introduction

Apart from the subject of straight normal flight and all the problems of stability and performance concerned with it, more thought has been given to the spin than to any other form of motion of an aeroplane. The reasons for this are not far to seek. The spin is the evolution which, until the recent introduction of certain well known methods of lateral control and stabilisation, invariably followed the loss of control at speeds below the stall, and its occurrence near the ground has been responsible for the majority of fatal flying accidents.

It is the only form of stable uncontrolled motion which is exhibited by an aeroplane other than that which it may possess in straight flight. This stability has in a number of cases proved extremely dangerous, if not fatal, as in prolonged spins on certain aeroplanes it has been found impossible to recover.

In spite of all the work that has been done the present state of knowledge of this evolution is very far from complete. The difficulties of research in spinning are many. The problem may be attacked in a number of different ways; by observation on models either in a wind tunnel or falling freely in air, by full-scale experiments on the actual aircraft, and by theoretical analysis. The practical difficulties of adapting the conventional wind tunnel to a study of spinning are very great, and free models only give very rough data. Full-scale experiment on the actual aeroplane is undoubtedly the best method, but like all full-scale work in flight, it is extremely expensive and very slow. There are also considerations of safety for the crew.

The problems of the prevention of accidental entry into the spin following the stalling of an aircraft have been solved very largely by the use of automatic wing tip slots, and inter-connected slot and aileron controls. There is, however, great need for further work not only on this side of the question, but more especially on the subject of the prolonged spin and safe recovery therefrom.

The present paper is a discussion of the position of work on the prolonged spin and an attempt to indicate the lines along which design should proceed to ensure safe recovery from this manœuvre.

The Prolonged Spin

There are many who hold that work on the prolonged spin serves no useful purpose. Their arguments are, generally speaking, that it is a useless manœuvre, one which no one would ever indulge in deliberately, and further, that it need never occur in ordinary flying accidentally.

The first of these arguments is probably sound; spinning is not particularly pleasant as a pastime, and appears to have little value from the military point of view, since the offensive weapons of a spinning aircraft are useless, and it is still open to attack from a pursuing enemy. As a deliberate manœuvre of commercial flying it can be dismissed at once. It is not true, however, to say that a prolonged spin need never occur. It may occur as a result of temporary unconsciousness of the military pilot through, say, the failure of oxygen at high

altitudes or in some other exigency of war flying. It may occur and has done so frequently to aircraft flying in dense clouds and fog.

A few words should be said about the physiological effect of spinning. If a spin takes place in dense cloud or under such conditions that the pilot has no visual reference for the attitude and path of his machine, he will not realise instantaneously that he is spinning, as he would normally. This means that possibly several turns will have taken place before he attempts recovery. The effect of the spin on his organs of balance, the semi-circular canals, will be such that, as soon as the rotation in the one direction has stopped, he will imagine that he is rotating in the other, and therefore there is every possibility that he may enter a second spin in the same direction as the first. He becomes more and more confused, and if there is still no visual reference the process may continue until the ground stops it permanently.

It will surely be agreed that no aeroplane can be termed safe, which may, under these or similar conditions, enter a state of flight in which the pilot, in addition to his other difficulties, finds that control over his aircraft is extremely small or even non-existent.

This brings one to a definition of the aircraft which has perfect safety in a prolonged spin. It is that aeroplane which will at any stage of a spin recover instantaneously[1] upon release of all controls, and return to normal stable flight with a minimum loss of height.

There are a few aircraft which would fall into this category, and then one has all the other classes; those that will recover of themselves but more slowly, those that require centralisation of controls, and those that require reversal of rudder, or stick full forward, or an application of both. A definite danger line is reached when the control column remains full back when released in the spin, and requires ever-increasing pressure to move it forward.

Finally come the aircraft which should have red ribbon tied to their tails, which develop some form of the so-called " flat spin " with high incidence, and very fast rate of rotation, and whose controls become entirely inoperative even under the persuasion of bursts of engine or other such artifices. The only cure for these is a parachute and it is interesting to observe that sometimes merely shaking the bottle in the face of the patient is sufficient, for there have been several cases in which such a spin has been stopped by the pilot preparing to leave with his parachute, whether by reason of some change of air flow over the tail, due to his raising himself in the seat, or through what cause it is difficult to say.

It should be observed that no aircraft has a definite motion which could be called *the* spin for that aeroplane. Spins vary in character for the same aircraft even when all such factors as centre of gravity position, tail setting, and control angles remain the same, and on most spins which have been carefully observed it has been found that there is generally a continual change of characteristics, the spin usually tending to become " flatter " as it proceeds.

The term " flat spin " has been used much recently and may be misleading. It is often intended to imply that an aeroplane has taken up a motion entirely different from its normal spin. On the other hand, it is also used by pilots to indicate that the spin, whilst retaining most of its ordinary characteristics, is such that the nose appears to rise and cover a very wide circle on the ground.

Used in the first sense it is probably rare, since most spins go on building up progressively. Used in the second sense, it must be applied with caution because whilst the nose of an aeroplane may appear to a pilot to be high in a spin, the motion may have none of those other characteristics usually associated with the term " flat," such as low rate of descent and low normal acceleration. In some instances a spin has been described as flat when calculation of the mean incidence has shown this to be comparatively low.

[1] Instantaneously may be taken to mean within say 1 turn.

Though most spins cannot be described as steady in the scientific sense there are some which appear unsteady even to the observer watching from the ground, and violent periodic changes of rate of rotation and incidence occur throughout the descent. These are usually somewhat unpleasant from the physiological point of view, though recovery from them is nearly always easy.

Research and Test Work on Spinning

Until comparatively recently the only organised research work bearing on the prolonged spin was that carried out at the National Physical Laboratory on models, and the full-scale experiments and model tests at the R.A.E., Farnborough. As a result, however, of dangerous spinning qualities discovered in a certain aircraft (now obsolete) whilst undergoing full-scale tests at Farnborough, prolonged spins were instituted as a routine test at Martlesham Heath for all single-seater and two-seater Service aircraft and also for those civil aeroplanes undergoing trials for an aerobatic certificate of airworthiness. The institution of these tests has turned the attention of constructors all over the country to the prolonged spin and much valuable data is being obtained.

Model Experiments

Model research on spinning may be carried out either in the wind tunnel or by means of free models dropped from a height. Both methods suffer from severe limitations. There is evidence to show that scale effect is serious in both cases. Due to the complicated nature of the motion it is very difficult to reproduce a spin in the wind tunnel, and although this method is invaluable for investigating such factors as autorotation, rolling, yawing and pitching moments, the effects of sideslip, and all the conditions likely to be met with in the spin, any deductions made as to the combination of all these factors assume that their effects are additive, a point which is very questionable. A great number of measurements have been made on the rolling balance which, as its name suggests, rotates the model at different speeds, whilst the couples set up in so doing may be measured. Much work has also been carried out using the ordinary three-force and three-moment balances.

An interesting suggestion put forward abroad and also in this country is that of a vertical tunnel in which the airstream is drawn upwards at such a speed that the model, though spinning freely, may remain at a constant height and thus render the observation of its motion comparatively easy.

Free flight model tests suffer from the difficulty of observation. The usual method is to take cinematograph films of the motion, and to analyse these, but this is not very satisfactory. Moreover, the length of the spin is limited by the height from which the model may be dropped. In spite of these difficulties, however, some interesting experiments have been carried out. The models are made of Balsa wood and are constructed so that their moments of inertia as well as their geometrical form are to scale. They are launched by means of a special gear to start the spinning motion and the controls are centralised or reversed after a given number of turns by means of a time fuse to investigate the recovery qualities. Precautions are taken for catching them in a large net to prevent destruction on hitting the ground.

Full-Scale Experiments

The first organised full-scale work on spinning was that carried out by Major F. W. Gooden, at Farnborough, on the "Spinning Stability of F.E.8.,"[2] although the first concise account of the motion was given as early as 1912 by

[2] Spinning Stability of F.E.8. Appendix I. R. & M. 618. "Investigation of the spin of an Aeroplane." Glauert. Lindemann and Harris.

Lieutenant Park, who spun an Avro biplane accidentally and recovered rather luckily by reversal of rudder only. Since Major Gooden's experiments, much work has been done on a number of aircraft, of which the most complete investigation so far carried out was that on the Bristol Fighter.[3]

In these experiments a standard Bristol Fighter was fitted with all the scientific equipment necessary for analysis of the spin. The instruments carried were as follows:—

1. Control movement recorders on rudder, elevators, and ailerons.
2. A single axis accelerometer arranged to measure the normal acceleration, fitted as near the centre of gravity as possible.
3. A three-axis gyroscopic recorder giving the rates of turn in roll, pitch and yaw.
4. An automatic observer, recording the position of longitudinal and lateral bubbles, aneroid, and stop-watch.

All the above were correlated by a timing circuit driven by an electric clock. In addition a tapping key was provided in the observer's cockpit which, when depressed, made a break in all the records simultaneously. Thus the observer could start the whole recording system in motion before entering the spin, and then when the first turn or two had been completed, and the spin had settled down, he could depress the key for an instant to denote the commencement of the steady portion, and again at the point when the pilot reversed controls for recovery.

To eliminate the airscrew effect, a band-brake was provided by which the pilot could lock the propeller in a fixed position.

An attempt was made to measure sideslip by means of yaw meters, but no accurate readings could be obtained from these, and the effort was therefore abandoned, and the values of this quantity and the angle of incidence calculated. It is singularly unfortunate that the sideslip is so difficult to measure directly in a spin. In early days it was thought to be small, and not of very great importance, but recent study indicates that it has a considerable bearing on the spinning characteristics.

The results of these experiments show that a number of turns elapse before the spin becomes steady. They also show the considerable variation which occurs in the spin of an aeroplane even under identical conditions as to loading and control positions.

Measurements of Moments of Inertia

Some mention should be made here of the measurement of the moments of inertia of an aircraft about its three principal axes,[4] the longitudinal, parallel to the wing chord, the lateral, parallel to the leading edge of the wing if there is no sweepback, and the normal to the wing chord.[4] These, of course, have their origin at the centre of gravity.

The measurement of the first two is carried out by swinging the whole aeroplane as a compound pendulum about an axis parallel to the axis concerned, and the moment of inertia about the normal axis is calculated from the results of torsional oscillation of the aeroplane on a bifilar suspension.[4] No doubt in time to come this will be a routine operation such as the determination of the centre of gravity, but at present it is in an experimental stage and some difficulties have been experienced in obtaining accurate results.

Much other full-scale experimental work has been carried out in this country and abroad, with the object of investigating the relative importance of the various

[3] Bristol Fighter. "Experiments on the Spinning of a Bristol Fighter Aeroplane." K. V. Wright. R. & M. 1261.

[4] See Appendix.

factors thought to have influence on the spinning characteristics. In America, particularly, several interesting series of tests have been made.

To obtain anything approaching complete quantitative data is extremely difficult without very elaborate equipment with consequent risk of breakdown. Qualitative data are not easy to procure, since the powers of observation of the crew of an aircraft are vitiated by the spinning motion and the height at which such experiments must be carried out in the interests of safety make ground observation difficult. The cinema camera has been of value in this direction.

Routine Tests of New Type Aircraft

All the immediate practical information required about a given spin can be obtained by the use of an aneroid and stop-watch in the hands of an experienced pilot. With these two instruments and careful observation, under admittedly difficult conditions can be obtained:—

1. The rate of rotation.
2. The rate of descent.
3. The height loss per turn.
4. The height required to regain level flight after the control movements have been made for recovery.

The addition of a recording accelerometer at the centre of gravity gives the normal force and an approximate value of the incidence attained.[5]

This brings one to the routine acceptance tests required by the Air Ministry on all single-seater and two-seater military aeroplanes and on civil aircraft seeking an aerobatic airworthiness certificate. The former are required to carry out spins of eight turns in either direction. The tests commence with the centre of gravity forward, and the tail incidence at its maximum (*i.e.*, tail adjustment full forward) and proceed to the case where the centre of gravity is on the aft limit and the tail adjustment is full back, on the assumption that these represent the worst conditions arising.

The pilot takes readings of height at entry, the number of turns in the spin,[6] the height at which he commences recovery, the height at which he regains level flight, and the turns executed after the recovery has been started. He also notes the methods of entry and recovery and any peculiarities of the spin.

Any aeroplane to pass these tests must recover in less than three turns after reversal of controls.

Civil aircraft have a less severe test of four turns in either direction.

Multi-engined aircraft are exempt from these tests, as it is considered that the distribution of the heavy masses along the wings would produce dangerous inertia couples in a prolonged spin. There are, however, cases on record of twin-engined aeroplanes having been spun for a turn or two only, and a safe recovery having been effected.

These routine tests are furnishing a steadily increasing volume of data which, when considered in conjunction with the more detailed information obtained from research sources, enable one to give a survey of those factors which have been shown to have influence on the spin.

Factors Affecting the Characteristics of a Spin

It was recently stated at a congress on spinning, held in America, that there were no less than eight hundred factors affecting the spin of an aeroplane, the greater part of which were entirely uninvestigated. It is, of course, impossible

[5] See Appendix.

[6] Where the edge of a wood, a river or some similar well defined line is not available for counting, the reflection of the sun as it strikes the lower plane is extremely useful for this purpose.

to deal here with more than the very few which have been shown to be of importance.

In order to study these factors it is necessary to form some fairly clear idea of the motion. For this purpose it is convenient to consider a spin first as it would be seen by an observer with glasses watching from the ground, that is with reference to external " fixed " axes; and secondly, as it would appear to the pilot of the aircraft, and as it would be registered by the instruments used in the full-scale research already described; that is, referring the motions, forces and couples to the " body " axes of the aeroplane.

The ground observer will notice that the aircraft is descending on a vertical spiral path of very small radius rotating about its own longitudinal axis as it does so. The attitude of the fuselage will be more or less steep according as the incidence in the spin is low or high. The leading edge of the wings will be nearly horizontal. The motion will appear uniform except in certain special cases, where he may notice that one part of each turn appears quicker than the rest.

The pilot of the aircraft will observe that the aircraft is rotating with considerable velocity in roll, and that provided there is no wind the nose of the aeroplane travels over the same points on the ground below on each successive circuit, taking between two and three seconds for each turn. The three-axis gyroscopes will show that there is a large rate of yaw, and a small and variable rate of pitch. There will be a rate of descent varying according to the wing loading and the incidence of the spin. On small civil types, training machines, and lightly loaded aircraft generally, this rate of descent varies between 5,000 and 6,000 feet per minute, but on some heavily loaded military aircraft which spin very steeply rough calculations appear to indicate a rate of descent as high as 8,000 or even 9,000 feet a minute. The pilot will notice a centrifugal acceleration holding him in his seat, but this will not be excessive and the accelerometer will show it to be between 1.0 g. and 3.0 g. If the spin is a " steady " one and not abnormal in any way, he will not experience any physical discomfort beyond the fact that difficulty will be found in re-focussing the eyes on any part of the aircraft, after watching the earth. Those not accustomed to prolonged spinning often experience extreme nausea from merely glancing down at the dashboard after timing turns on some fixed point on the ground, and watching the tail or a wing tip is not to be recommended. The pilot may also find that the aircraft appears to spin one wing down, and that he is forced to one side of the cockpit. This indicates sideslip.

If very high incidences are reached the circle covered by the nose will increase, and the pilot will probably suffer from dizziness, due to the excessive rate of rotation. The centrifugal force will decrease under these conditions. Considerable pressure will probably now be required to move the control column forward for recovery.

If the spin is jerky, it will be uncomfortable, due mainly to the variation in the pitch and normal acceleration.

Summarising, the following are characteristic features of a spin:—

1. Wing incidence anything between stalling and 90°.
2. An approximately helical path of the centre of gravity of the aeroplane about the vertical axis of the spin.
3. Moderate centrifugal acceleration depending on the radius of the helix, which again depends on incidence.
4. High rate of descent decreasing as the angle of incidence increases.
5. Considerable rates of rotation in roll and yaw and a small rate of pitch.
6. Sideslip.

It will be noticed that the first item in this summary draws attention to the fact that the incidence is greater than stalling. This is fundamental to any spin, and the first factor to be discussed arises entirely from this circumstance.

1 *Autorotation*

Although the spinning characteristics of a complete aeroplane bear but little resemblance to the autorotative characteristics of its wing unit tested alone as a model in a wind tunnel, it is true to say that without the autorotative property possessed by the wings, spinning would not occur.

It is well known that any typical wing, whether monoplane or biplane, is stable in roll below the stalling angle, but that if this incidence is exceeded the wing becomes unstable in roll, and if disturbed will start rotating, the speed settling down to a steady value depending on the incidence and speed of the relative airstream.

The autorotative characteristics of a wing at any given incidence depend upon the slope of the lift curve and the value of the drag coefficient at that incidence. The necessary condition that autorotation takes place is that

$$dk_L/d\alpha + k_D < 0$$ [7]

It will be seen at once that since k_D is always positive, this condition demands a negative slope of the lift curve. Also since k_D increases rapidly above the stall, an increasingly rapid fall off of lift is required to maintain autorotation. The equation also explains why the wing possessing a sharp-peaked lift curve is more likely to have sudden spinning qualities at the stall than one possessing a flat-topped lift curve. In this connection, however, it is of interest to note that the very wings which are safe in this respect at angles immediately above the stall are those on which large autorotative couples are likely to develop at high angles of incidence, and which therefore may prove dangerous in a prolonged spin.

In every lift curve there is usually a wave between 20° and 40° in which the slope becomes very much less negative or may even be positive in value for a few degrees. This wave terminates the first region of autorotation, and if a positive slope occurs, the wing will pass on with increase of incidence through a period of latent autorotation in which the wing will be stable unless started rotating artificially to a given rate. If no positive slope occurs the wing will pass straight to a region of no autorotation, after a longer region of true autorotation.

In the case of monoplanes and biplanes with high stagger and large value of gap/chord ratio, the rapid increase of k_D now prevents any further autorotation, but in the case of biplanes with gap/chord ratios of less than 1, and particularly those with zero or negative stagger, the value of k_D is very much less at high angles of incidence and the latent autorotation region will almost certainly be followed by a second region of true autorotation which will extend to very high angles of incidence with extremely high rates of roll. These were the characteristics possessed by the Springbok and the B.A.T. Bantam, both of which aircraft had fatally dangerous spins.

In the actual spin one of the most important influences on the rolling motion is the sideslip, which produces a moment tending to raise that wing tip towards which the aircraft is sideslipping. It has been stated[8] that " 20° of sideslip in the direction aiding rotation will approximately double the rolling moment and extend the range of autorotation up to 90° while the same amount in the other direction will normally overcome the original rotation." Thus, if there is outward sideslip in a spin, increased autorotation may be expected, whilst inward sideslip will have the reverse effect.

It will be seen from the form of the expression for rolling moment[9] that the autorotation speed is directly proportional to the forward velocity and inversely proportional to the span. These relationships have been confirmed experimentally, and from this one may expect that in free flight at any given incidence at which autorotation takes place, the angular velocity will be decreased approxi-

[7] See Appendix.
[8] " The present Status of Research on Aeroplane Spinning." F. E. Weick.
[9] See Appendix.

mately directly as the wing span is increased and as the square root of the wing loading is decreased.

The effect of tapering and twisting a wing to give " wash-out " at the tips is to delay the stall to a higher mean angle of incidence without, however, any change in the value of the maximum lift coefficient. This will reduce the likelihood of accidental spin, but it is not clear what the influence will be at the higher angles of incidence. In some tests on a twisted and tapered symmetrical section the effect was to increase the values of rate of roll in the first range which was only slightly delayed in comparison with the corresponding range for the untwisted rectilinear aerofoil, and to remove a second range altogether.

The effect of slots at very high angles of incidence is another point on which research is required. Since the maximum lift coefficient is increased by their use, the negative slope of the lift curve after stalling must of necessity be increased also. The slots should also decrease the value of K_D at the higher incidences, and it is safe to say that little or no help towards decreasing autorotation tendencies can be expected on theoretical grounds from slots at incidences well beyond the stall.

Having considered the main contribution of the wings alone, let us build up the aircraft further. The body in itself has but small aerodynamic effect, though the influence of its shape on the action of the tail unit is very great. Let us, therefore, add the body and tail unit in one step.

The Tail Unit

In considering the tail unit of a spinning aeroplane it is obvious that the air flow over it is completely unsymmetrical with reference to its principal axes. It is, therefore, impossible to consider the vertical surfaces without at the same time taking into account tailplane and elevator.

First take the case of a longitudinally stable aeroplane trimmed to fly level at some speed above the stalling speed. The condition of stability demands that the horizontal tail surface is at an angle of incidence less than that of the wings. The flow over the tail will be symmetrical apart from a slight twist due to slipstream, and there will be little or no shielding of the control surfaces by the body or by each other. The control angles will be zero, or nearly so.

Now consider the conditions when the aircraft has just entered a spin. The elevators will be hard up, and the rudder hard over in the direction to assist the spin. If the joystick is released at this stage it will move forward and the aircraft tend to dive out of the spin. The attitude of the aircraft, however, with reference to the line of flight will be such that the tail plane is already rendered less effective than in normal flight, and the fin and rudder are partially shielded by the body. There will be a component of outward sideslip at the tail, due to the yawing of the whole aeroplane about the centre of gravity, which will tend to make the tail plane still less effective since the inside half will become partially shielded by the body. The effect of moderate outward sideslip on the rudder from the point of view of damping the rate of yaw will be slight but beneficial.

If the rate of yaw and the incidence increase to high values, these conditions will be much accentuated and the load on the elevators reversed so that the stick, when released, moves back and considerable force is required to move it forward.

Thus, whereas the tail controls may be more than powerful enough to put the aircraft into a spin from straight flight, their effectiveness once the spin is established is a very different matter. If the rate of yaw is very high and they are small and badly situated behind a short bluff fuselage, it is easy to see that the elevator may be quite incapable of providing the necessary pitching moment to reduce the incidence to less than stalling, and the rudder incapable of making any reduction in the rate of yaw.

A similar state of affairs exists with regard to the setting of the tail plane, for while it is possible to obtain a spin with higher incidence by setting the tail adjustment further back, it is common knowledge amongst pilots that winding it forward once that spin is established has little or no effect on the rate of rotation.

When controls tend to become inoperative in this way it is, of course, useless to apply them gently for recovery. The only effective method is to reverse them suddenly and fiercely and hold them firmly in their reversed positions for several turns possibly before any response is noticeable.

This suggests that the method of using the controls at entry to the spin may affect its characteristics and this has been shown more than once. Apart, however, from the fact that violent application tends to produce a higher rate of rotation (a result which might be anticipated), nothing further has been established.

As a result of their investigations on the subject of the action of controls in the spin the Aeronautical Research Committee suggested that for safety the rudder volume coefficient[10] should not be less than 0.05, and later the Design Sub-Committee suggested a figure not less than 0.08 for the " effective volume " of fin and rudder.[10]

It is evident that careful interpretation of these rules is needed on the part of the designer. Apart from the fact that the surfaces must be disposed so that they will not be blanked seriously by the body when spinning, it is of the utmost importance that the coefficients are not obtained by providing very large control areas in conjunction with a very short fuselage. The American rule that the centre of pressure of the tail organs should not lie inside the semi-circle described on the wing span as diameter guards against this possibility. Again, from the point of view of preventing a fast spin, care should be taken that an ample proportion of area goes to the fixed surfaces, and that the limiting control angles are not greater than they need be for adequate manœuvrability in normal and (where the need arises) for aerobatic flight.

The Lateral Control

Beyond their action in starting a spin, where their adverse yawing moment plays a large part, normal ailerons have little effect. It is true that the rate of rotation may be slightly lowered by putting the control column over towards the inside of the spin, and raised by moving it to the outside, but this is the limit of their action on a normal aircraft.

The effect of using the inter-connected slot and aileron control has been tried out, but in the case where the spin had already developed, beyond reducing the angle of bank when the stick was put hard over against the spin they produced no other response. This might be expected, since the main features of this control are the elimination of the powerful adverse yawing moment of a normal aileron in stalled flight and the improvement of the control in roll.

Floating ailerons, ailerons with excessive angular movement, and other special forms of lateral control have been tried, sometimes with success on the model. In full-scale form, however, they have proved either ineffective or else have had some serious mechanical difficulty.

Centre of Gravity Position

It is a generally accepted fact that an aeroplane may be made more and more unsatisfactory from the point of view of recovery from a spin by moving the centre of gravity progressively aft. No hard and fast rule can be laid down for a safe aft limit as it depends on the aerodynamic and other characteristics of the aircraft, but trouble may develop if it is farther back than 0.30 of the mean

[10] See Appendix.

aerodynamic chord. Perhaps the most important way in which the centre of gravity position affects the spin is through the degree of longitudinal stability of the aircraft. If the centre of gravity is forward of the mean centre of pressure the aircraft will be stable and there will be a pitching moment tending to depress the nose and decrease the incidence when stalled. If the centre of gravity is behind the mean centre of pressure (and it will be remembered that the latter tends to move forward at high angles of incidence), then the aircraft is unstable and when stalled the pitching moment will be in the sense to raise the nose still further when spinning.

This question of longitudinal stability, as affected by the centre of gravity position, is perhaps more important when considering the accidental or incipient spin. It is obvious that the longitudinally stable aeroplane will be safer in this respect at slow speeds than one which is unstable. Difficulty in preventing the centre of gravity from taking up a far back position is experienced mainly in the design of commercial aircraft and certain types of reconnaissance machines where cameras and wireless gear must be readily accessible to the rear observer. In these types, longitudinal instability is particularly undesirable apart from considerations of safety in spins.

In general the position of the centre of gravity does not affect the balance of the pitching couples in a spin to any great extent of itself. Its importance lies in the secondary effect it has on the growth of incidence where, owing to the peculiar characteristics often associated with the autorotation of a wing, a small change in pitching couples may be responsible for a large change in the rate of spin. On the other hand, there are cases on record where this does not occur and the centre of gravity may be moved with perfect safety from the spinning point of view to a very far aft position. Considering autorotation qualities alone, it seems likely that monoplanes might possess this happy quality, but the limited spinning experience obtained so far with this type has not indicated that it is borne out in practice.

In the present state of knowledge, however, there is no doubt that the further forward the centre of gravity is, the better from every point of view, bearing in mind that the general manœuvrability of the aircraft and in particular its ability to make three point landings must not be impaired.

The Inertia Couple

The effect of mass distribution on spinning qualities has been receiving much attention recently both in this country and America. In the case of the single-engined aeroplane, the principal masses are nearly all placed on the fore and aft axis, and variation is limited to the longitudinal distribution on this line. In the case of certain military types, however, bombs may be carried at some distance out from the fuselage under the wings, whilst the increasing popularity of the multi-engine aircraft for commercial use makes the question of lateral distribution of considerable interest also. At the same time it must be admitted at the outset, that from the little experience which has been gained so far on the spinning of multi-engined aircraft, the prospects of a safe recovery from a prolonged spin on such a type would appear extremely doubtful.

Numerically, the major component of the inertia couple is that about the lateral axis of the aircraft and, making the assumption that the axis of the spin is in the plane of symmetry of the aircraft, its value is approximately

$$\tfrac{1}{2}(C-A)\,\Omega^2 \sin 2\alpha \ ^{11}$$

It is, unfortunately, always in a sense to raise the nose of the aeroplane since C, the moment of inertia about the normal axis is greater than A, the moment of inertia about the longitudinal axis, for all normal aeroplanes.

[11] See Appendix.

The above expression varies as the square of the resultant rotation in the spin (Ω), and for a constant value of Ω reaches a maximum when α the incidence in the spin reaches 45°.

Since it varies directly as the difference $(C-A)$, it is evident that placing large masses at the extremities of the fuselage on the longitudinal axis will increase its value, since by so doing C will be increased whilst A will not be affected. Thus, a wide distribution of the various loads at large distances from the centre of gravity is likely to be harmful from the spinning point of view. But although it has been shown that the inertia pitching couple may become comparable in magnitude with the aerodynamic pitching couples in the spin, there are on record experiments carried out, where increase of the moments of inertia about the lateral and normal axes, involving an increase of $(C-A)$ of nearly 150 per cent. produced no appreciable effect on the spinning characteristics, nor on the ease of recovery.[12]

In this case, however, the value of $(A-B)$ was changed by -150 per cent. at the same time, which in turn would change the value of the inertia yawing couple by the same amount, and it can only be concluded that this change may in some way counteract the effect of the original change of $(C-A)$, in spite of the very small value of the inertia yawing couple compared with the pitching couple. The counteraction may take place through the medium of sideslip or some other factor.

As may be imagined, increasing the moments of inertia about the longitudinal and normal axes by placing masses along the wings has a very definite and harmful action in a spin, and it was found in the same series of experiments referred to above that an unstaggered biplane could be rendered dangerous comparatively easily by this means, and the weights had to be released before recovery could be effected. On the other hand, the same alteration produced but little effect on a biplane with stagger and sweepback, and one can only conclude here that more liberties may be taken with the loading of an aircraft whose aerodynamic qualities are good from the spinning aspect, than with one in which they are not.

The only recommendation to the designer that can be made at present, to guard against possible danger from the inertia point of view, is the very obvious one that all large masses should be placed as near the centre of gravity as it is possible to have them.

The Airscrew

If the airscrew is rotating during the spin it exerts a gyroscopic couple which acts about the lateral axis of the aircraft. Its numerical value[13] is directly proportional to the moment of inertia of the airscrew, its rate of rotation, the rate of rotation of the aircraft in the spin, and sin α. Thus, here again we have a couple increasing as the incidence and rate of spin increase. In this case, however, the couple is not invariably in the direction to raise the nose of the aircraft as its sign depends on the direction of rotation in the spin relative to that of the airscrew. If these are in the same sense then the nose of the aircraft will be depressed, and if in the opposite, it will be elevated.

It has been shown[14] that the value of this gyroscopic couple, due to airscrew, is roughly one-sixth of the inertia couple, due to the aircraft itself with engine throttled back, and that it may rise to twice or three times that amount with full power depending on the airscrew r.p.m. These values, it will be noted, apply to airscrew alone. In the case of an aeroplane fitted with a rotary engine,

[12] "Army Air Corps Experiments on Tail Spins." H. A. Sutton. American Society of Automotive Engineers.

[13] See Appendix.

[14] "The Spinning of Aeroplanes." Gates & Bryant. R. & M. 1001.

there is an additional large couple due to the engine itself. The effect of this couple will be remembered on the Camel when it caused considerable difficulty to inexperienced pilots even in normal flight on steep turns.

Now that the rotary engine has fallen from favour the gyroscopic couple is mainly that due to airscrew alone. Though small it is of interest for two reasons.

From the research point of view it is responsible for a great part of the dissimilarity which is nearly always found between the spins in opposite directions of any aircraft. It has been eliminated in some cases by locking the airscrew as described earlier in this paper.

From the practical point of view it has considerable bearing on the question of the use of power for recovering from spins. In some cases where recovery has proved impossible by other means it has been effected by the use of engine either steadily for a period, or in bursts.

If the airscrew is rotating in the same sense as the spin then application of power can only have a beneficial effect on recovery, for the gyroscopic couple will tend to depress the nose, and the slipstream will increase the efficiency of the tail organs, though it will be remembered that it will not strike them in the favourable manner of straight flight. On the other hand, if the airscrew rotation and the spin are in the opposite sense, then the effect of opening up the engine to effect recovery may be doubtful, for the gyroscopic couple may more than undo the work of the slipstream.

In the few cases on record where spins were stopped by the use of engine it is very difficult to tell to which of the two factors the recovery was mainly due. It can only be emphasised that the characteristics of the spin are very sensitive to small changes in the pitching couple, and it is to this fact that the gyroscopic action of the airscrew owes its importance.

Conclusion

In this paper has been discussed a number of factors bearing on the characteristics of the spin of an aeroplane. There are many others, and it may be that future research will point out some of these as being of even greater importance. There is certainly much to be done before the prolonged spin is completely understood, and before it will be possible to predict accurately from the design of a given aircraft whether its spinning characteristics will be satisfactory or not. Certain features have, however, been proved definitely harmful and the designer will do well to avoid all of these if he wishes to make sure of his aircraft. The interaction of one factor on another in the spin has been shown repeatedly and is a lesson to be borne well in mind.

The author wishes to thank the Air Ministry for permission to read this paper.

He also wishes to take entire responsibility for all statements made and any opinions expressed.

APPENDIX I.

(a) Measurement of Moments of Inertia

The N.A.C.A. have published details of their methods of measuring the three moments of inertia of an aeroplane in technical note No. 265.[15] These methods have proved more satisfactory than any others tried so far and have been adopted elsewhere. Even so the agreement between results as measured and as calculated is not very good in all cases, and further development is required.

For measurement of the moment of inertia (A) about the XX axis, the aeroplane is placed on suitable suspensions so that it may be swung about a parallel

[15] " Measurement of Moments of Inertia of Full-scale Aeroplanes." M. W. Green.

axis. The time of oscillation is observed and the moment of inertia deduced from the two general equations:—

$$K = \sqrt{(gh/4\pi^2n^2 - h^2)} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{and } MK^2 = I \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

K = radius of gyration.
g = acceleration due to gravity.
h = pendulum length (pivot to C.G.).
n = oscillations per second.
M = mass of the aeroplane.
I = moment of inertia (in this case A).

A similar method is used for measuring B, the moment of inertia about the YY axis, the suspensions being arranged to allow of oscillation about an axis parallel to YY. Similar equations are used for calculating this moment of inertia, substituting appropriate values.

For measurement of the moment of inertia C about the ZZ axis, owing to the difficulty of suspending the aircraft suitably for swinging about an axis parallel to ZZ the same bifilar suspension as for measurement of B is used, and the aircraft oscillated as a torsional pendulum.

The moment of inertia is calculated by using in place of the previous equation (1)

$$K = (a/2\pi n)\sqrt{(g/l)} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where K = radius of gyration (as before).
a = horizontal distance from the C.G. to the points of attachment of the bifilar suspensions.
n = oscillations per second.
l = length of the bifilar suspensions.
g = acceleration due to gravity.

The technical note lays stress on the fact that the oscillations must be small, otherwise the results are vitiated by damping, due to the surfaces of the aeroplane. Attention is also drawn to the fact that great care must be taken to eliminate friction from the pivots.

NOTE.—It has been assumed in this paper that the principal axes of the aeroplane are the chord axes. This is not necessarily correct, but is sufficiently accurate to a first approximation.

(b) *Mean Angle of Incidence in a Spin*

The mean angle of incidence in a spin can be obtained approximately from the value of the normal acceleration as recorded by an accelerometer placed at the centre of gravity.

$$\alpha \text{ (mean)} = \sin^{-1}(l/a)$$

where a = normal acceleration in multiples of g.

The assumptions made in the development of this formula are (1) that the rate of rotation in pitch is zero, and (2) that the resultant acceleration is perpendicular to the chord.[16]

(c) *Criteria for Autorotation*

The necessary condition for autorotation is that the slope of the curve of normal force on the aerofoil referred to wind axes, should be negative at the point corresponding to the mean angle of incidence.

This leads to the well-known condition

$$dk_L/d\alpha + k_D < 0$$

where $dk_L/d\alpha$ is the slope of the lift/incidence curve at the mean angle of attack of the aerofoil. (See any textbook on Aerodynamics.)

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[16] R. & M. 1261. "Experiments on the Spinning of a Bristol Fighter Aeroplane." K. V. Wright, B.A.

It should be noted that the above expression is only a criterion for true autorotation. It does not apply for the occurrence of latent autorotation.

Using strip theory, the value of the rolling couple on the aerofoil causing autorotation is given by

$$L=\rho V^2 c\int_o^{S_o} S\delta k_L dS$$

where C = the chord of the aerofoil.
S_o = semi-span of the aerofoil.
δk_L = difference in lift coefficient between a pair of strip elements at a distance S from the axis of rotation on the centre line of the aerofoil.

The expression may be written

$$L=(\rho V^4 c/p^2)\int_o^{pS_o/V}(pS/V)\,\delta k_L d\,(pS/V)\ ^{17}$$

For steady rotation $L=0$; p = angular velocity in radians per sec.

(d) *Rudder Volume Coefficient*

The rudder volume coefficient is defined as

$$A'' \times l''/Ss$$

where A'' = area of rudder.
l'' = distance of C.P. of rudder from C.G. of the aircraft.
S = wing area.
s = semi-span.

The effective volume of fin and rudder is obtained by (1) substituting for A'' above, the following area:—

The side area of the fuselage for a distance $l'/2$ forward from the C.P. of fin and rudder plus the area of fin and rudder less that area blanked off above and behind the tail plane and elevators between boundary lines drawn upwards and backwards at an angle of 45° to the chord of the tail plane from the leading and trailing edges of that unit.

(2) l', the distance of the C.P. of fin and rudder combined, from the C.G. of the aircraft,[18] being substituted for l'' above.

(e) *The Inertia Couple in the Spin*

The components of the inertia couple about the principal axes of the aeroplane are[19]

Rolling $L'=(B-C)\,q'r'$ . . . . . (1)
Pitching $M=(C-A)\,r'p'$ . . . . . (2)
Yawing $N'=(A-B)\,p'q'$ . . . . . (3)

where p' is the rate of roll referred to chord axes.
q' ,, ,, pitch ,, ,, ,,
r' ,, ,, yaw ,, ,, ,,

(f) *The Gyroscopic Couple due to Airscrew*

The gyroscopic couples due to the inertia of the airscrew about chord axes of the aeroplane are[19]

Rolling $L'_1=0$.
Pitching $M_1=-A_1\omega\Omega\sin\alpha$.
Yawing $N'_1=A_1\omega\Omega\cos\theta\sin\phi$.

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[17] "Applied Aerodynamics." L. Bairstow.
[18] Aircraft Design Memo. 131.
[19] "The Spinning of Aeroplanes." Gates & Bryant. R. & M. 1001.

Where ω is the angular velocity of the airscrew considered large in comparison with Ω.

A_1 is the moment of inertia of the airscrew about its axis.

On the assumption that θ is nearly 90°, it is evident that the pitching couple about the lateral axis is the only one of importance.[20]

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MODERN DEVELOPMENTS IN THE HELICOPTER

BY

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The problem of the helicopter is one which has intrigued inventors for centuries, and which has perhaps attracted more attention and effort than any other phase of design of heavier-than-air craft.

Even as far back as the fifteenth century we find Leonardo da Vinci devoting several years of his life to studying bird and mechanical flight, and he has left many sketches showing flying machines. Of particular interest to us is a design showing an aircraft consisting of a sustaining screw driven about a vertical axis.

The first helicopter actually to fly was a small model, little more than a toy, which was shown before the French Academie des Sciences in 1784 by Launay and Bienvenue, and since that time literally thousands of schemes have been proposed by inventors all over the world. The patent files of all countries are full of helicopter specifications, the greater part of them being based on faulty physical principles, and obviously impracticable.

The true helicopter is a heavier-than-air craft, which is able to lift itself and some useful load off the ground in still air, which can hover over a given spot, can descend vertically under its own power and can make a safe descent in the event of engine failure. In addition, it must be able to move horizontally at will at a satisfactory speed, be controllable and stable under all flight conditions. The possibilities for practical use of such a craft would appear far-reaching, and certainly have fired the imagination of layman and technician. The advantages of being able to take-off and land without large specially prepared aerodromes are obvious, and the type, provided its special characteristics can be obtained without paying too big a price in performance and pay load, might well prove of great assistance in the development of civil aviation.

It is in the required characteristics mentioned above, namely, horizontal translation, stability and control, that the real difficulties of the problem lie, and few of the earlier inventors in the field appear to have seen the problem as a whole. In most cases they appear to have considered only the requirement of obtaining sufficient lift to sustain the load. As long ago as 1904 Colonel Renard read papers before the Academie in which he developed the theory of sustaining screws to quite considerable length, and formulated a correct relationship between minimum horse-power required, weight to be lifted and airscrew dimensions. To-day, with modern theoretical knowledge, any competent airscrew designer can provide a rotating system providing adequate lift in hovering flight.

Let us now look at the two special problems of horizontal translation and stability.

Translation.—In horizontal motion with a lifting screw system the difficulty arises that the blade which is at any instant advancing into the relative wind has a higher air speed than the retreating blade on the opposite side, and therefore experiences greater lift. This inequality of lift gives rise to an inconvenient overturning couple. In all the early examples of rotating wing helicopters it

will be noticed that they have invariably had pairs of screws either superimposed on a common axis, or side by side and rotating in opposite senses. This twin disposition was to overcome the unbalanced forces on the single screw in translation. It also has the merit of balancing out the torque reaction which with a single driven screw would result in the fuselage turning about the screw shaft. The expedient is therefore satisfactory so far.

Stability.—Most people are familiar with the flying wheel which is an elementary helicopter and rises in the air when spun rapidly by an archimedian drill device. When the wheel is made to rotate it lifts itself and climbs or hovers apparently quite stably, until the speed of rotation falls and the lift dies away. The single rotating screw is indeed stable for practical purposes and its stability is due partly to the gyroscopic action of the rotating mass and partly to aerodynamic forces. The gyroscopic forces are of course such that if the axis of rotation pitches in a certain plane, a torque is produced in the plane at right angles and the axis precesses. The radius of the precessional motion of the axis is generally quite small and the screw appears quite stable.

The aerodynamic forces act in a very similar way. If the axis of the screw inclines there will be a component of the thrust in the direction of the tilt, and the screw moves in that direction. Under horizontal translation, this motion results in a lack of balance of the lift forces causing a couple in the plane at right angles to the initial inclination, which deflects the axis in that plane. This deflection causes a side velocity in the new plane followed by a moment in the plane at right angles which restores the original position of the axis. The result is that the tip of the screw axis describes a circular path about the vertical exactly as caused by the gyroscopic couples.

Unfortunately, these righting forces, both gyroscopic and aerodynamic, disappear when there are two screws working in opposite directions, so that so far we are still without a solution to our problem.

There have, of course, in the past been several helicopter designs which have been able to lift themselves and have made translatory flights with some degree of success, and I now wish to give you a rapid review of some of them, partly to show the lines along which earlier workers approached the problem, but chiefly to pave the way to a better understanding of the modern types.

Oemichen

One of the earliest workers to obtain practical full-scale results was Oemichen, who was an engineer associated with the Peugeot works.

In his first helicopter, the sustaining screws were two in number, each two-bladers, 20.8 feet in diameter and driven in opposite directions by a two-cylinder engine developing about 25 h.p. The total weight was 748lbs. and the apparatus failed to rise. A small ballonet was therefore added, partly to give the necessary extra lift of 154lbs., but also, as Oemichen appears to have realised the need, to stabilise the machine.

With the ballonet, a number of elementary flights were made, the maximum height reached being 5 metres and distance in horizontal flight 60 metres.

In Oemichen's next aircraft he had four principal lifting screws, and five subsidiary screws with variable pitch. Two of these latter, placed one either side, were for propulsion and the other three were used as controls, their pitch control mechanism being operated from the control column and rudder bar, and their axes such as to cause movement about the appropriate aircraft axis.

The lifting screws were two of 21 feet diameter and two of 24.4 feet diameter, and the engine a 120 h.p. Le Rhone. The loaded weight was 2,200lbs.

This helicopter is historic in that it carried out several flights of from 600 to 700 metres, hovered for five minutes over a fixed point and finally was the first, in May, 1923, to cover an officially observed closed circuit of 1 km.

No attempt was made to cope with descent in the event of engine failure and all flights were at about 1 to 1½ metres from the ground. The horizontal speed was of the order of 25 km./hr. (15 m.p.h.).

Pescara

One of the best known workers on the helicopter was Pescara. His second aircraft, built in 1924, had a rotating system consisting of two superimposed rotors, each with four biplane blades driven in opposite directions through a clutch and gearing by a Hispano 150 h.p. engine.

The diameter of the rotors was 23.4 feet and the aircraft weighed 1,870lbs. The method of control is interesting. The biplane blades were mounted so that they could be rotated in order to vary their incidence.

A wheel on the control column, if turned to the left, increased the incidence of all the blades on the screw turning to the right and decreased the incidence of the other set of blades. In this way a difference in the torque reaction was produced causing the fuselage to turn to the left or vice versa.

For the other controls, the cables were taken from the control column in such a way as to decrease the incidence of the blades as they passed through the plane of the column in its displaced direction, and increase on the other side. The shaft was thus caused to tilt in the direction in which the column was inclined. A separate lever was provided simultaneously controlling the incidence of all blades.

Horizontal flight was achieved by tilting the axis and utilising the horizontal component of the screw thrust. Several flights were carried out, but the gears were a continual source of trouble, breaking after only a short time of running, and the craft was hopelessly unstable. It, however, carried out horizontal flights up to 700 yards and up to ten minutes duration.

Berliner

Another type to which I should like to refer in passing is the Berliner, because of its control system.

The aircraft was built around a Nieuport Scout by replacing the wings by a pair of 14ft. 10in. diameter screws, one on each side. The tail surfaces were standard except that a bigger rudder was fitted and for the addition of a small variable pitch screw rotating in the horizontal plane. The pitch of this screw was controlled simultaneously with the elevator and served to make fore and aft control effective whilst hovering. Lateral control was effected by means of shutters in the down draft of the lifting screws.

The craft weighed 1,250lbs. empty and 1,350lbs. loaded and the engine was a 110 h.p. Le Rhone.

Horizontal translation was effected by tilting the screw axes.

In official trials the aircraft rose 7ft. and traversed 100 yards at a speed of about 15 m.p.h.

Karman Petrosczy

This helicopter was proposed during the late war to replace kite balloons for observation purposes. Its construction was carried out under the direction of Professor Karman, the well-known aerodynamic technician, and the lifting screws were designed by Herr Asboth.

It consisted essentially of three 120 h.p. Le Rhone engines driving two superimposed wooden screws in opposite directions. No control was provided and stability was assured by a three-cable mooring system.

The weight was 3,520lbs. of which 660lbs. was useful load and the screws were 19.8 feet diameter.

It rose to heights up to 150ft., staying up for considerable periods, the maximum being one hour.

Moineau

Whilst the lifting screw idea has always held the field, there have been a large number of inventors who have proposed to get vertical lift by aerofoils mounted as the blades of a paddle wheel. In this type, of which the Moineau is representative, the blade system rotates about a horizontal axis, and the incidence of the blades at the various points of their travel is controlled by an eccentric linkage. In the Moineau, the relative eccentricity of the eccentric shaft and driving member is variable so as to put the line of total reaction under the control of the pilot. Means were also provided to vary differentially the eccentricity of the front and rear ends of the blades so as to give lateral control. One would suggest that progress along these lines is impossible. The lifting efficiency must be low due to the fact that during part of the cycle, the blades do not contribute anything to lift, whilst still taking power to drive them. The control forces available are small, and are derived at the expense of lift, since the total reaction is substantially constant, and stability is, of course, non-existent.

Ornithopters

The other main class of direct lifting aircraft is the ornithopter, of which no successful examples have been produced.

Modern Progress

It will be seen from this survey that whilst some success had been achieved up to 1924, the main problems were still untouched. Great impetus has, however, been given to the helicopter movement by Cierva's development of the gyroplane, which possesses three features of vital importance to the helicopter:—

(1) The difficulty due to the lateral overturning moments of a single screw in translation has been discussed. Cierva devised the exceedingly neat and effective plan of hinging the blades so that they could rise or fall independently under the influence of the air loads on them. All lack of balance due to forward motion is thus simply and adequately overcome.

(2) The rotating system is perfectly stable.

(3) The blades being freely hinged are not subjected to heavy bending moments, nor are the loads on the blades greatly influenced by acceleration, since the centrifugal loads are so much greater than the air loads.

The blade structure is in fact mainly subjected to tensile loads, and an important truth not yet generally realised is that the structure weight of the hinged-blade rotating wing aircraft tends to decrease with size instead of to increase, as is the case with the conventional aeroplane. It may thus eventually be possible to build big helicopters more economically as regards structure than big aircraft.

(4) It has paved the way to safe forced landings by helicopters by showing the high parachutal value of the auto-rotating wing.

Isacco Helicogyro

M. Isacco, who has had long experience on helicopters as engineer to Pescara, must be given the credit of first realising the great contribution which Cierva had made to the cause of direct lifting aircraft, and he immediately applied it to his helicogyre, which must, I think, be considered as being the first helicopter project to show promise of giving a complete solution to the problem.

M. Isacco has himself given a description of the principle on which he is working in a paper before the Royal Aeronautical Society so that I need not go into great detail. It will be recalled, however, that in his machine each blade is virtually a complete small monoplane, freely hinged but constrained to move in a circular path, and provided with a complete set of aerodynamic controls. The power units are fitted on the blades near the tips. This course was adopted for two reasons. First, because of the high weight attendant on the use of gearing and its proneness to give trouble, due to the high torque which has to be transmitted, and secondly, because in hinged blade wings the coning angle of the blades must be limited, and to do this the centrifugal loads must be kept sufficiently high. There is thus a limiting weight for the blades dictated by reasons quite distinct from structural strength. It therefore pays to put power unit weight into the blades.

In helicogyres forward motion is achieved by a separate front power unit and the axis is normally kept vertical. This is a more efficient way of obtaining the necessary horizontal thrust than tilting and using the horizontal component of total reaction.

The first machine was designed as a single seater and had only two blades, 41 feet diameter, each with a Cherub engine driving its own airscrew.

I have had the privilege of carrying out trials on this aircraft, and can say that, with my weight, the aircraft, then weighing 1,320lb., was just capable of supporting itself at ground level, the wing engines developing 60 h.p. and the wing system r.p.m. being 55. I should point out that the aircraft was never intended to lift itself to considerable heights in true vertical ascent. M. Isacco prefers to put only sufficient horse-power into his blades to give substantially full lift near the ground, relying on the big increase in lifting efficiency due to small relative wind to get him off with a very short run. I was not able to attempt a free flight in the open, but carried out taxying trials in a large airship shed. With quite a small forward velocity I was able to get the aircraft airborne, and it proved to be fully stable and controllable.

Under all conditions the aerodynamic control of the blade incidence was satisfactory and the blades themselves were quite stable.

The Air Ministry has built a helicogyre to M. Isacco's design. This differs considerably from its prototype.

(1) It is a two seater and has four blades in its rotating system.

(2) The rotor diameter has been increased to 49 feet, partly from a desire to obtain true vertical ascent in still air and partly because of the increases in weight beyond M. Isacco's original design due to additions and strengthening up to British standards.

(3) It will be realised that, although the blade incidence could be differentially controlled, as in the Pescara system, we could not derive any rolling moment in this way, since the blades are freely winged. Whilst satisfied that in the air the craft would be stable, it was none the less considered necessary to provide definite lateral control in order to give the pilot full command whilst landing through bumpy air. This was achieved by adding a supplementary control rotor. This control rotor carries four planes rigidly attached to the central part, but whose incidence is controlled from the pilot's column in exactly the same way as in the Pescara system.

The aircraft as built weighs 2,420lb., and just lifts itself at about 45 rotor r.p.m., the wing engines developing 120 h.p. In horizontal flight its performance should be satisfactory. Aerodynamically I believe the aircraft to be satisfactory, but very considerable trouble has been experienced with the wing engines. The idea of putting the power units in the wings is, I think, fundamentally sound, but it brings new troubles in its train in that the carburation, distribution and oiling

systems are operating in an intense centrifugal field. These troubles are accentuated in the actual aircraft, since the only engines available were horizontal opposed twins, so that distribution to the outer cylinder is faulty and it tends to oil up. An air-cooled vertical in line engine would doubtless be satisfactory as regards distribution and lubrication, but it may be that a new type of carburettor will have to be developed. The only other criticism that one can offer is that the aircraft is mechanically complicated, in that there are four sets of engine controls and ignition controls, four blade aileron controls, four control rotor controls, and four trimming surface controls to be brought through the rotating centre. In spite of this complexity the rotating system and the central part are relatively neat.

Neither of the existing aircraft has yet flown, but by close attention to piston clearance, the fitting of high pressure scraper rings, and metering the fuel through a centrifugally controlled orifice, the blade engines are now running fairly satisfactorily, at any rate for short periods, and I hope that before long the type will show that it can take the air satisfactorily.

Curtiss Bleecker

The Curtiss Company of America have devoted four years' research to the helicopter, the work being under the direction of a young technician, Bleecker.

It consists essentially of a rotating system of four blades of 47ft. 4in. diameter, each blade having about 80 square feet of surface. The blades are driven each by its own airscrew mounted forward of the leading edge at about nine feet radius, and these screws are driven through gears and shafting from a wasp engine mounted with its plane horizontal. The cooling of the engine in itself presented a major problem, and a special study involving a year's work was carried out on cooling and cowling design.

With this method of drive it is, of course, not possible to use the hinged blade system, but you will have realised that the Cierva system is a simple and neat method of suitably varying blade incidence throughout the revolution. This same incidence variation is achieved in the Bleecker in the following way. Each blade is provided with its own elevator, and the blades being free to rotate in the pitching sense their incidence is controlled by these elevators to give the correct instantaneous incidence throughout the revolution. The elevators are also connected to the control column in such a way as to give differential incidence control for lateral and longitudinal control purposes, and the mean incidence of all blades can also be controlled by a separate lever.

Directional control is provided by a rudder. Horizontal flight is to be achieved by tilting the axis.

Fully loaded the aircraft weighs 3,400lb.

D'Ascanio

The Italian helicopter invented by D'Ascanio created history by being the first aircraft to take the record of the Fédération Internationale in the helicopter class. The following performances are officially recognised by the F.A.I.:

Vertical ascent to 20ft., remaining stationary for 1½ mins. and then landing.
A straight flight of 560 metres at a height of from three to seven metres.
Ascent to 18 metres in 1.40 secs.
Duration flight of 8¾ mins.
A closed circuit flight over a 1 km. course, landing at the starting point.

The lifting system consists of two 39ft. diameter superimposed co-axial rotors, each of two blades.

The blades, each 50 sq. ft. in area, are free to flap about a horizontal hinge on the central shafts, and can also rotate about a large diameter ball bearing, so as to change their angle of incidence. They are stabilised by a tail plane of about 3 sq. ft. area, carried behind the outer tip. An elevator of small chord is fitted to the tail planes, and it is through this that the pilot controls the blade incidence to suit the various conditions of flight.

The two blade systems are driven in opposite directions by a Fiat 50 a.s. engine developing 95 h.p. at 2,000 r.p.m. The drive is carried through a clutch and gear box to a vertical shaft, at the head of which is a differential gear. The internal shaft carries the central pinion of an epicyclic gear, and the planet wheel bearings are attached to the exterior tubular shaft which carries the lower blades. The outer ring of the gear carries the upper blades.

A simple ratchet gear is provided to let the lifting system continue in rotation in the event of engine failure.

To control the aircraft the fuselage carries three small control screws, which are driven from the lower gear box so that they are always operative, even in case of engine failure, so long as the lifting system is rotating. The one at the rear rotates in the horizontal plane and is for longitudinal control, that on the pilot's left also rotates in the horizontal plane for lateral control, and that on the right in the vertical plane is for directional control. All these screws are of variable pitch type, the pitch being controlled from the joy stick.

The aircraft weighs 1,676lb., and lifts itself at 1,800 r.p.m. of the engine, which is then developing about 65 h.p.

According to eye witnesses the aircraft rises and descends vertically and lands gently. It can remain stationary or travel horizontally at will. It is perfectly stable and controllable and free from unbalanced forces and vibrations. Its speed and rate of climb are very small, but except for this it would appear to meet all fundamental requirements of perfect horizontal flight. These statements are borne out by the film which I have seen of the record flights.

Asboth

The last helicopter to which I wish to refer is the Asboth, designed by Herr Asboth, to whom I have already referred in connection with the design of the lifting screws of the Karman Petroczy tethered helicopter.

Herr Asboth during the late war was director of the Austro-Hungarian State airscrew factory, and in connection with the Karman Petroczy helicopter undertook research work on lifting screws.

In the course of this and subsequent work he made and tested over a thousand model airscrews, in series in which he changed one parameter at a time. The mass of data derived from his tests has been condensed into a few dozen graphs and tables and some 80 formulæ. In using these charts all that it is necessary to know is the weight to be lifted and the available power, and the best diameter, blade width, blade angles at all radii, top and bottom camber of the blade sections, etc., all follow directly. The system of design is thus highly simplified, and in practice definitely gives the desired screw characteristics. The Asboth screws, designed in this way, give very high lifting efficiency.

The inventor, unfortunately, is in reduced circumstances, so that the actual aircraft which he has built is somewhat crude in its construction. It is none the less a most successful piece of apparatus.

It consists essentially of a steel tubular framework acting as a support for the power unit and lifting screws. This frame also carries the control organs and serves to accommodate the pilot.

There is no undercarriage in this prototype aircraft, the body tubes being simply extended to form legs ending in hemispherical cups into which four ordinary inflated footballs are strapped.

The sustaining screws are, to the eye, two ordinary wooden two-bladed airscrews, each 14 feet diameter, driven in opposite directions by a Clerget engine through a simple gear box and concentric shafts. The screws revolve at half engine speed, *i.e.*, about 650 r.p.m. at full throttle.

Control is provided by a series of six vertical flaps in the downwash of the lifting screws. These flaps are coupled to a wheel control column and rudder bar in such a way that the pilot's control movements are exactly the same as in a conventional aeroplane. For instance, pushing the column forward raises the two front flaps and the nose is depressed.

Originally the aircraft was fitted with a front tractor variable pitch airscrew, but with this the aircraft was liable to land with too much forward speed, with which the elementary undercarriage could not cope. At present, therefore, and until a castoring wheel undercarriage can be provided, the horizontal speed is deliberately reduced by tilting the screw axis for translation.

I have carried out personal trials on this aircraft. My trials were all free flights in the open and show remarkable results. The aircraft takes-off vertically with a surprising rate of climb of the order of 300 feet per minute, hovers indefinitely at any height, and is fully controllable.

The manœuvrability is of a low order, the controls though effective being relatively slow in response. They are, however, quite adequate for ordinary manœuvres.

The most important feature, and one contrary to what would be expected from my earlier remarks, is that the aircraft was stable about all axes. My trials were all carried out in still air so that the effect of bumps could not be examined, but on displacing the aircraft from equilibrium no divergence was started, and it returned to an even keel. It was indeed possible to abandon all controls when hovering.

Landing is extremely simple and even with the elementary undercarriage is literally " as light as a feather."

The only features lacking in the early example is provision for safe descent in the event of engine failure. Herr Asboth has, however, satisfied himself that his screws will remain in autorotation and that a safe rate of descent can be achieved.

The details of the aircraft are:—

Screw diameter	14 feet.
Flying weight	1,080lb.
H.P.	About 110 at 1,300 r.p.m.
Blade area each blade	8.5 sq. ft.
Disc loading	6.84 lb./sq. ft.

During my trials the minimum r.p.m. required for hovering at a height of about 20 feet was 1,050. The engine horse-power is problematical as the engine was old and running hot, but 85 h.p. would appear to be close to the truth.

Because of its extreme simplicity and the results attained this helicopter must be looked upon as one of the most interesting yet built.

In conclusion, I desire to express my thanks to the Director of Scientific Research for his permission to read this paper and to Herr Asboth and Count Cippico for their permission to describe the Asboth and D'Ascanio aircraft.

APPENDIX

So far I have not touched on the subject of lifting screws mathematically, and as an appendix wish quickly to indicate the lifting efficiency which is to be expected. For a complete analysis I would refer the reader to Mr. Glauert's two valuable papers, R. and M.'s 1132 and 1157.

It is convenient to express the thrust and torque of a lifting screw by two non-dimensional coefficients T_c and Q_c defined by:—

$$T = \text{thrust} = T_c \pi R^2 e \Omega^2 R^2.$$
$$Q = \text{torque} = Q_c \eta R^2 e \Omega^2 R^3.$$

R is the radius of the screw and Ω the angular velocity. The power absorbed is of course ΩQ, but it is important to note that this is the power actually transmitted to the screw, so that the efficiency of the drive must be taken into account.

The merit of a sustaining screw may be expressed in the form

$$M = T_c^{3/2} / Q_c = \sqrt{(T^3 / \eta R^2 e \Omega^2 Q^2)}.$$

Similarly, if W be the total weight of the helicopter and $\eta\rho$ the power available for driving the screw it is convenient to write

$$M = (W / \eta\rho) \sqrt{(W / \eta R^2 e)}.$$

Sustentation will be possible if $N = M$ and we can therefore write:—

lb./h.p. which can be lifted at ground level $= (M \times \eta) / \sqrt{(w \times .0375)}$

where w is the disc loading in lb./sq. foot.

If the h.p. available is such that at maximum h.p. the lb./h.p. are less than this the helicopter will climb and its ceiling will be given by the following table:—

Power ratio.	Ceiling.
1.29	5,000ft.
1.69	10,000ft.
2.23	15,000ft.
2.98	20,000ft.

For a lifting screw of the flapping blade type with the ratio blade area/disc area equal to 0.25 the optimum value of M is 1.16.

The form of the expressions given above shows that economy of power is obtained by using a lightly loaded screw, and that the vertical ascent of a helicopter depends critically on the disc loading.

A low disc loading is also of importance in the forced landing since the vertical velocity in autorotational descent with engine stopped will be of the order of $21\sqrt{w}$.

A point of great importance which must be kept in mind is the great increase in lifting efficiency when the screw is in horizontal translation.

As an example, assuming a ratio of blade area/disc area equal to 0.25, and with disc loading of 1.0 lbs./sq. ft., the figure of merit for a hinged blade system rises as in the following table:—

Horizontal velocity.	Figure of merit M.
Static	1.16
15 ft./sec.	1.44
30 ft./sec.	2.02
45 ft./sec.	2.52
60 ft./sec.	2.88
75 ft./sec.	3.14

Another feature which demands careful attention in comparing the tested performance of helicopters is the big influence of proximity to the ground. Within a height comparable with the screw diameter the lifting effect of the screw is considerably increased. With a 40ft. diameter screw, for instance, the weight which can be sustained near the ground is about 15 per cent. greater than at a height of 40ft. This ground effect was very noticeable on my trials of the Asboth helicopter. On taking off with minimum horse-power required the aircraft rises to about 15ft. and will not climb beyond this until more throttle is given.

FUELS AND MODERN METHODS OF TESTING

BY

H. C. TETT (ANGLO-AMERICAN OIL COMPANY)

(*Lecture read before the Gloucester Branch.*)

Petroleum as it occurs in Nature is a very complex mixture of hydrocarbons. Imagine a mixture of petrol, paraffin, gas oil, fuel oil and lubricating oil, together with a certain amount of water and other foreign matter, and you have an idea of crude petroleum as it is found in Nature.

It is interesting to note that practically the sole use of petroleum for some time after the first well was drilled was as a burning oil. The object of the refiner was therefore to produce as large as possible a percentage of paraffin from his crude. The lighter fractions were regarded as a necessary evil, and it became a very difficult problem to dispose of them satisfactorily. It was, of course, necessary to keep the flash point of the paraffin high enough to avoid the danger of explosion of the lamps in which it was used. Petroleum inspectors were employed to ensure that the paraffin on sale was not " adulterated " with the then valueless light petroleum spirit. No market other than as a source for lighting gas was found for this light spirit until the advent of the motor car.

The early motor car had rather an inefficient carburettor, usually of the " surface " type, and, in addition, the refiner at this time still wanted to squeeze as much as he could of his crude into the valuable kerosene fraction. It is then obvious why petrol at that time consisted of very light and low boiling fractions. As the motor vehicle increased in numbers petrol became more valuable, carburation was improved, and for these reasons it was possible and indeed desirable that the distillation range of petrol should be increased to include some of the heavier fractions.

Petrol has all the qualities necessary to make the ideal fuel for the spark ignition internal combustion engine except one, it has a tendency to knock under certain conditions.

If one gallon of petrol were burned completely and all the available energy produced could be used it would be sufficient to drive an ordinary 15 h.p. car along a level road for about 400 miles, or to raise it through a vertical height of nearly 10 miles. It will be obvious from this that when petrol is used in an internal combustion engine a large percentage of the available energy is wasted. This loss can be divided into two—heat loss and frictional loss, both to some extent unavoidable. Considering the first only—we can increase the efficiency of the engine by raising the compression ratio. But when the compression pressure is raised petrol unfortunately begins to knock, or detonate. The efficiency of our engine is therefore limited by the tendency of the fuel to knock.

I have mentioned this at this stage as I now intend to describe to you the usual tests employed in the examination of petrol and their significance. It will be obvious that a reliable test to determine the tendency of a fuel to knock is all-important. For this reason I shall devote somewhat more attention to mechanical methods of testing than to chemical analysis.

The tests applied may be divided into two groups—(*a*) *Chemical and Physical* and (*b*) *Mechanical.*

(a) Chemical and Physical Tests

(a) In the first group the following tests will be taken in the order given :— Specific Gravity, Distillation, Chemical Analysis, Gum, Sulphur, Colour and Calorific Value.

Specific Gravity is measured very simply by the use of the hydrometer, a correction for temperature of course being applied. This test tells us very little indeed of the quality of the spirit unless the crude from which it is refined and the refining process are known. Actually, of course, the gravity of the petrol determines the level of the petrol in the carburettor float chamber, and therefore the level of the petrol in the jet. If large variations occurred it would be necessary to make adjustments on changing from one fuel to another.

In addition to this, if two fuels are equal in all other respects and one has a higher specific gravity than the other, it is possible to obtain more miles per gallon from the heavier fuel, although not more miles per pound. The variations in the gravity of petrols found on the English market are not large, certainly not large enough to affect the operation of the motor car engine to any noticeable extent, and the test is therefore of little imporance.

Distillation is a test for volatility. The apparatus used is of standard dimensions and consists simply of a flask fitted with a thermometer and having a side arm leading to a condenser tube surrounded by water kept at zero centigrade by the addition of ice; 100 c.c.'s of the fuel being tested are placed in the flask which is then connected to the condenser and heated gently under standard conditions. After a little time the petrol commences to boil, the vapour given off is driven over and condensed, and the first drop falls into the receiving flask. The temperature recorded by the thermometer when the first drop appears is called the " initial boiling point." The petrol continues to boil at a steady rate, and as the lighter fractions pass over and the fractions with higher boiling points are vaporised, so the temperature rises. The thermometer is read at suitable intervals, usually when the distillate in the graduated receiving flask has reached 10%, 20%, 30%, and so on. The temperature recorded when the flask boils dry is called the " final boiling point."

The conditions under which the test is made govern the rate of heating— which should be such that the first drop of distillate appears after not less than five and not more than ten minutes, distillation then proceeding at a rate of 4-5 c.c.'s per minute, or approximately two drops per second. The position of the thermometer in the flask and all other details of the apparatus are also carefully defined so that comparable results can be obtained by different observers and on any standard apparatus.

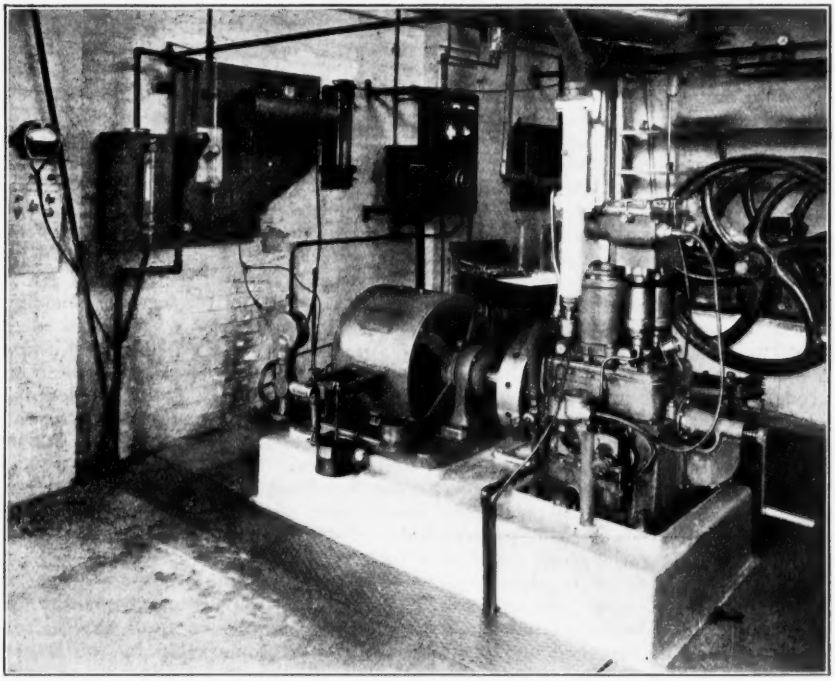
The distillation test is second in importance only to the test for anti-knock value. It determines mainly the ability of a petrol to satisfy the following three requirements. Easy starting, rapid acceleration, absence of crankcase dilution.

To obtain easy starting it is necessary that when the engine is quite cold and turning slowly, sufficient petrol must evaporate to produce an explosive mixture in the cylinder. Mixtures of petrol vapour and air are explosive only over the range of proportions from 1.5% to 6% of petrol by volume or from 1 : 18 to 1 : 5 by weight. So to ensure easy starting petrol must contain a sufficiently high proportion of low boiling point fractions, as these are the most volatile or easily vaporised constituents. To satisfy the distillation test in this respect a petrol must therefore show a low initial boiling point, and what is more important, a reasonably high percentage distilling below say 70°C. In hot countries trouble is sometimes experienced owing to the petrol being too volatile and causing a gas leak in the carburettor. This difficulty is seldom met in this country, any complaint being usually the reverse, namely, that a petrol is insufficiently volatile to get easy starting in cold weather.

Good acceleration depends largely, of course, on the anti-knock value of a petrol as it necessitates maximum power at low engine speeds, conditions under which knocking usually occurs. It is also necessary that the fuel shall vaporise rapidly and as completely as possible so that the engine will respond readily when the throttle is opened.

Crankcase dilution of lubricating oil is caused by the high boiling fractions of a petrol with a high final boiling point, which probably remain unburnt in the cylinder and so pass into the crankcase and contaminate the oil.

It is thought by some people that petrol is completely vaporised when it enters the cylinder. The author is more inclined to agree with the view that



minute drops consisting of the higher boiling fractions remain even at the high temperature existing in the cylinder. It can certainly be proved that a petrol having a high final boiling point will cause dilution of crankcase oil, impairing its viscosity and lubricating value. For this reason the end point should never be higher than 200°C., particularly as comparatively few of the cars operating in this country are provided with heated inlet manifolds.

The easy starting properties of a fuel also affect crankcase dilution to some extent and should be such that excessive use of the air strangler is unnecessary. It is, of course, common knowledge that a fuel of poor volatility will require either the use of the strangler or a very rich carburettor setting to obtain smooth running from cold. With the strangler in use a considerable amount of liquid petrol is sucked into the cylinder, some of it fails to evaporate and eventually reaches the crankcase having meanwhile prevented the efficient lubrication of the

cylinder walls at the very time when it is most needed, the oil being cold and very viscous and therefore circulating very slowly.

Chemical Analysis was, until a few years ago, confined to determination of the aromatic content of a fuel. It was considered that the aromatic content was almost entirely responsible for the absence or otherwise of the tendency to knock. The aromatics are the members of the hydrocarbon series falling into a certain chemical class. Examples are benzene, xylene, and toluene, of which ordinary benzol is a mixture. Benzol consists therefore entirely of aromatics and, as you know, is added to petrol to improve its anti-knock qualities.

It is now known that other classes of hydrocarbons such as naphthenes and olefines also have a high anti-knock value, and a petrol may have a very low aromatic content and yet possess a good anti-knock value. The member of the naphthene series known as cyclo-hexane was prepared in America during the War, and it was found that it could be used with a compression ratio as high as 12 : 1.

It is impossible here to go into the details of the various chemical methods of evaluating the percentage of these various groups in petrol. A large number of methods exist and the reliability of most of them is a matter of opinion so that it would require a separate paper to deal with the subject at any length.

The proportion of each chemical class, paraffins, naphthenes, aromatics, in a " straight run " petrol, that is a petrol produced by ordinary distillation from crude, depends on the crude from which it was refined. Olefines are found in petrols produced by the process of cracking, and are often prone to oxidation, causing discoloration of the fuel and a tendency to deposit gum. This gum, when present in undesirable proportions, deposits on inlet valves, causing sticking and consequently erratic running and burning of the valves.

This trouble of the instability of cracked petrol is, however, gradually being overcome and an increasingly large percentage of the world's supply is now produced by cracking, a much larger proportion of petrol being obtainable from crude by this method than by straight distillation. The process of cracking consists, briefly, of distillation at high temperature and pressure, under which the heavier molecules are split up into lighter ones. Thus, by heating gas oil or even lubricating oil under these conditions a product having the boiling range required for petrol can be produced. In practice, of course, the less valuable constituents of crude such as gas oil and fuel oil are subjected to the cracking process to produce the more valuable petrol.

The Gum Test, as its name implies, is used to determine the tendency of a petrol to deposit gum. The test usually consists of evaporating a quantity of petrol under standard conditions in a glass or porcelain dish, the residue after evaporation being weighed and examined. The petrol marketed by the larger companies in this country seldom contains more that 2 or 3 milligrams of gum per 100 c.c.'s. In petrol containing a light percentage of cracked spirit the gum content may be as high as 20 or 30 mgms. per 100 c.c.'s, and cases have been met where as much as 500 mgms. per 100 c.c.'s were deposited. It should be understood that the various refining and cracking processes vary largely in the stability of the fuel produced, and it is possible to produce a cracked spirit having comparatively little tendency to form gum. In the case of aviation petrol the specification is very rigid as regards gum.

The Sulphur Test is a simple chemical method of evaluating the total sulphur present. It involves burning a weighed quantity of petrol in a special lamp and dissolving and estimating the sulphur dioxide thus produced.

A corrosive test is also carried out to detect the presence of active sulphur compounds in the acidic form. The quantity of sulphur present in the average British petrol is very low indeed, probably very much lower than is actually necessary, American specifications being somewhat more lax in this respect.

The Colour Test is carried out on an instrument named the " colourimeter,"

and is of little importance, most petrols being water white, although this does not seem to be really necessary.

The Calorific Value is determined by the use of a bomb calorimeter. A weighed quantity of fuel is exploded in a sealed container and the rise in temperature noted, results being expressed as British thermal units per pound. This determination is seldom made as almost all petrols have very nearly the same calorific value.

All these tests are, of course, applicable to Commercial, No. 1 and Aviation Petrols, the results to be expected differing in each case. Commercial petrols are of lower volatility and lower anti-knock value than the No. 1 grade, and aviation has a very much lower final boiling point than other petrols, usually about 150°C. instead of, say, 190°C.

There is no rigid specification in force for petrol in this country except, of course, the specifications imposed by certain buyers, such as the Air Ministry. A petrol may be sold as No. 1 and yet be very inferior in all respects to the No. 3 grade sold by reputable companies. This, of course, is a very undesirable, although unavoidable, state of affairs, and the only remedy for the consumer is to buy only from those companies whose reputation can be trusted, and who carefully control and maintain the quality of their grades.

(*b*) *Mechanical Tests*

We now come to the most important test of all, that carried out in an engine under actual working conditions.

Until comparatively recent years no satisfactory engine method of testing fuels was designed. The road test is subject to too many variations in external conditions beyond the control of the operator to be of any great value by itself, although it can serve a useful purpose as an additional test when carried out under carefully controlled conditions by an experienced observer.

The first engine fuel tests of value were probably made on engines fitted with variable compression combustion heads, such as the " Thornycroft " and " Ricardo E.35."

The method in this case is to raise the compression ratio while the engine is running, other conditions being fixed, until audible detonation occurs. The compression ratio at which this occurs is called Highest Useful Compression Ratio or " H.U.C.R." Obviously, if one petrol has a higher H.U.C.R. than another its anti-knock value is superior, and the results obtained from this method are truly comparative, but the actual figures obtained apply only to the particular engine upon which the test is made.

For use in routine testing of large numbers of samples the variable compression engine has one or two disadvantages, the greatest being that results depend to some extent on the observer, and two observers are quite likely to disagree as to what constitutes audible detonation. Also it is not easy to maintain identical conditions in testing successive fuels as each test occupies some considerable time.

This method of testing, however, has been of great value in research work as the power, fuel and oil consumption, etc., can all be very accurately determined by means of the apparatus usually supplied with these engines, such as the swinging field dynamometer and special fuel measuring devices.

In the laboratories of the Anglo-American Oil Company a " Thornycroft " variable compression engine has been installed for some years and has been of very great value.

With the recent increased importance of the anti-knock value of petrols, however, some reliable routine test which could be carried out by an observer not necessarily skilled in engine research has become absolutely necessary, and about three years ago the Anglo-American Oil Company installed an engine apparatus of special form embodying the Midgley Bouncing Pin.

As a considerable amount of work has been published in connection with the H.U.C.R. method of testing and very little in connection with the bouncing pin method, I propose to deal with the latter at some length.

A normal single cylinder overhead valve engine is used with a compression ratio high enough to cause knocking on any fuel which may require to be tested.

The essential part is Midgley's Bouncing Pin apparatus, which consists of a hollow cylinder screwed into the cylinder head and closed at its lower end by a light diaphragm fixed by a clamping ring. The bouncing pin itself is a loose fit inside the hollow cylinder and rests on the diaphragm, the top of the pin just projecting above the top of the cylinder. Immediately above the pin are a pair of contacts, both spring mounted, and with a gap of about .025in. The upper contact works against a spring stop just above it. Now if the pressure in the cylinder rises gradually the pin will rise as the diaphragm is deflected, but will remain in contact with it. If the pressure rise is sudden, as with detonation, the pin receives a sharp blow, and travels farther than the diaphragm, the extent of its travel depending on the severity of the blow and therefore on the intensity of the detonation. As long as detonation is absent the pin does not travel far enough to close the contact, but when detonation takes place contact is established, the duration of each contact depending on the intensity of detonation. The contact is connected in series with a battery, a switch, an electrolytic cell and a lamp. The electrolytic cell is filled with dilute sulphuric acid and current passing between its electrodes therefore generates gas which is collected in a graduated arm. The gas generated is proportional to the current and the time for which it flows, and is therefore proportional to the intensity of detonation. The lamp affords a visible indication of detonation.

The engines used in these testing sets are built up from Delco 1,250 watt lighting sets, modified very considerably, the power being absorbed by the Delco generator, and the current generated by an armature resistance. A variable field resistance on the control panel provides a means of adjusting the load and therefore the speed.

An improvement on the original water cooling has recently been introduced in the form of a reflex condenser providing evaporative cooling so that with water as the cooling medium the head is maintained at exactly 100°C. This is most important as recent experiments show that the temperature of the combustion chamber has a marked effect on detonation and on the relative value of various anti-detonating agents.

The original engines were provided with a very simple fuel supply system consisting of two carburettor cups fitted with constant level overflow pipes instead of float chambers. These were supplied, one with a standard fuel and the other with the fuel under test by pipes leading from overhead containers, the quantity being adjusted by needle valves to be just sufficient to maintain a steady overflow from the cups and so a constant level.

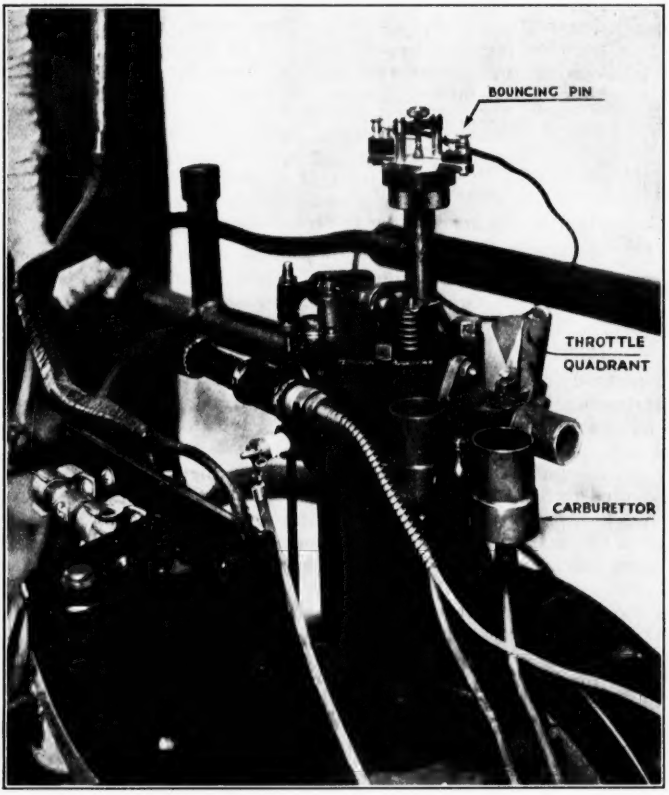
These cups have recently been replaced by variable level float chambers so that mixture strength may be adjusted for maximum knock on any fuel.

A three-way cock is provided so that the engine may be run on either fuel at will and changed over immediately from one to the other while running.

The control board also carries a distant reading thermometer to indicate the temperature of the cooling water, an electric tachometer or revolution counter to show engine speed, and an accurate stop-clock. A switch is also provided so that the battery used for starting purposes may be charged.

The method of conducting a test is as follows: The engine is run for about half an hour to obtain constant temperature conditions. The speed is then maintained absolutely constant, and the amount of gas generated in 60 seconds is measured with the engine running on the standard fuel, which is usually of higher anti-knock value than the fuel on test. The fuel on test is then supplied by turning the three-way cock and the amount of gas generated in 60 seconds

is again observed. This gives an indication of the relative anti-knock value of the two fuels. Ethyl fluid, the anti-knock component of ethyl petrol, is then mixed with the test fuel until it is equal in anti-knock value to the standard fuel. The proportion required is seldom more than one part in six hundred, and so this addition alters the test fuel only in respect of anti-knock value. When the test fuel is considered by the observer to have been made equal to the standard, several observations of the gas generation are made, first on one fuel, then on



the other. Successive readings should then be almost identical, and the two fuels are known to be equal in anti-knock value under conditions which are exactly the same for each.

Results are then expressed in the quantity of ethyl fluid required to bring the test fuel up to a given standard.

Fresh supplies of the standard fuel can be made up from time to time by matching the new supply with the old on the engine.

It will be observed that the beauty of this method lies in the fact that it is a null method and depends only to a small extent on the skill of the observer. The quantity of fluid added can be measured to any required accuracy, and the

gas generation can be repeated again and again until constant readings are obtained.

The bouncing pin method of testing was introduced to this country by the Anglo-American Oil Company, and has since been adopted by the Universities of Oxford and Cambridge, and also by the Air Ministry.

The various methods of applying engine tests to fuels may be summed up as follows:—

1. An engine must be used which can be made to detonate.
2. The engine must be under control from the non-detonating to the detonating range, which is accomplished by controlling the compression pressure by
   (*a*) Altering the compression ratio at fixed throttle opening.
   (*b*) Altering the throttle opening at fixed compression ratio.
3. Detection of detonation by
   (*a*) Audibility.
   (*b*) Bouncing pin.
   (*c*) Maximum power output.
4. Evaluation of results may be in terms of
   (*a*) Compression ratio, power output, or manifold vacuum.
   (*b*) An equivalent anti-knock fuel made by mixing an anti-detonating agent with a reference fuel of low anti-knock value.
   (*c*) The addition of an anti-detonating agent to the test fuel to equal a reference fuel of high anti-knock value.

The method just described is a combination of (2) *b*, (3) *b* and (4) *c*, and of course depends largely on the use of ethyl fluid as an anti-knock agent, which has the advantage that it does not alter the gravity or distillation range of the petrol to any appreciable extent.

The active component of ethyl fluid is tetra-ethyl lead, a compound which was discovered to have a remarkable anti-knock effect by Midgley in 1921. The original work was carried out in the research laboratories of General Motors, Ltd., and in the search for a suitable chemical no less than 30,000 different compounds were investigated. Several other compounds were found to have a pronounced anti-knock effect, but none was found to approach tetra-ethyl lead which is 600 times more effective than benzol.

It was soon realised that the advantages of a fuel containing such a powerful anti-knock agent would be of great commercial value and considerable further investigation was carried out, as a result of which a company was formed by the Standard Oil Company, of New Jersey, and General Motors, for the purpose of marketing ethyl fluid. As it is sold for blending with petrol, ethyl fluid contains about 60% tetra-ethyl lead, 40% ethylene dibromide, and a trace of red dye. The ethylene dibromide is added as a halogen bearer to convert the lead into lead bromide, which is inert and in every way harmless. The dye was added purely so that ethyl petrol might be distinguished from others.

The concentration of the fluid in ethyl petrol never exceeds one part in 1,300, and it has now been well proved by both the British and American Governments that no harmful results are possible from its use. As is usual with any new product, many troubles were attributed to ethyl when it was first introduced, both in America and in this country, but it is interesting to note that prejudice in this country has now almost completely disappeared, and in America the alleged harmful effects of ethyl on engine parts and public health, which at one time occupied so much attention, are now completely forgotten.

Manufacturers in America have taken advantage of the general distribution of ethyl by raising the compression ratios of their engines and so increasing their performance. Also in this country compression ratios have increased consider-

ably in the last two years, following the general improvement in the anti-knock value of fuels on the British market which resulted from the introduction of ethyl.

There is no doubt that the discovery of ethyl was the greatest development made in connection with the internal combustion engine in recent years.

From the following figures showing indicated horse-power for various compression ratios it is obvious that a great saving of fuel can be effected by using high compression ratios :—

Compression ratio.	Indicated horse-power.	Indicated thermal efficiency.
4	30.3	28.8
5	33.1	32.8
6	35.2	35.9
7	36.8	38.3

There has been a very marked tendency during recent years toward higher compression ratios, particularly in high performance cars and in aircraft engines. I have no doubt that the future will see still further developments in this direction, and the anti-knock value of fuels is therefore bound to play a very important part in future development of the internal combustion engine.

THE METHOD OF "LEAST WORK" AND THE STRESSING OF AEROPLANE STRUCTURES

BY

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(*Paper read before the Students' Section, December, 1929. Abridged.*)

INTRODUCTION

Object of Paper

In aeronautical engineering the strength-weight ratio is so important, apart from the fact that the consequences of structural failure are generally more disastrous, that it becomes essential to investigate the loads and stresses in the various parts of the structures far more accurately and in much greater detail in order that smaller factors of safety may be used.

It is often possible, in the case of structures of a low order of redundancy which have to be designed to meet several different systems of loading, to select by inspection those members which may be considered to be the redundant ones, the elimination of which would result in an increase of the design loads in most of the remaining members. If the members are designed to such increased loads many members of the structure will be overstrong and unnecessarily heavy. The excess of strength of the various members of the complete redundant structure will almost certainly differ, so that the weight of the whole will be increased above the essential in a greater proportion than the strength. Thus the return for the increased structure weight in the form of an increased factor of safety is uncertain and possibly non-existent, while, even if it does exist, it is not required if the true factor of safety desirable is known. This is the most favourable result of designing a redundant structure as if it were non-redundant.

However, it frequently happens that the effect of the presence of the redundant members (especially in space frames of irregular geometry) is so to modify the distribution of the loads throughout the structure as to increase considerably the design loads of some parts of the structure while reducing those of other parts, even though the structure is designed for several different systems of loading. (When there is only one system of loading for the structure, this inevitably occurs). Thus it is apparent that in most cases it is necessary to take into account the effect of any redundant members in a structure in order to ensure that the required factor of safety is attained.

From the points of view of both weight and strength it is, therefore, highly desirable that the presence of redundant members in all main parts of aircraft structures should not be neglected in the strength calculations.

Scope and Arrangement of Paper

No attempt is made to set out in full all the strain-energy arguments, proofs and formulæ which are available for determining deflections and for solving statically-indeterminate structures. These are to be found in a number of text-books.

THEORY

It is intended to consider chiefly those aspects of the subject most directly applicable to the more usual problems met with in the stressing of aeroplane structures, and this leads to a tendency to consider the subject with the framed structure nearly always in mind. It must be realised, however, that the method is of great value in the treatment of " incomplete " structures which rely on stiff-joints for stability, and other problems in which bending of the members is of primary importance.

It is not considered necessary to give here a complete series of definitions. It should be sufficient to mention that a " *pin-jointed framed structure* " is one that is built up of a number of bars or " members," each of which is joined, at its ends only, to some of the other members, and in such a way that no bending moment can be transmitted. Provided the external loading is applied at the joints only, the members are subjected only to direct loads. If there are only just sufficient members to maintain the geometry of the structure, the latter is " *simply stiff* "; if less than this, it is " *incomplete* "; if more, it is " *redundant.*"

Condition for Simple Stiffness of a Pin-jointed Framed Structure

The members of a *plane frame* all lie in one plane and can only resist forces in their plane. Consider the building-up of such a frame so that it shall be simply stiff. The simplest plane frame consists of three members joined in the form of a triangle. If it is desired to locate another point relative to this structure, two more members will be necessary and sufficient. Similarly, for each additional joint another pair of members will be required. If m is the number of members and j the number of joints, then, for simple stiffness,

$$m = 2j - 3.$$

If the number of members in a structure of this type is greater than m as given by this expression the structure is redundant, and the excess is the number of redundant members.

The members of a *space frame* do not lie only in one plane, and, provided the structure is not incomplete, the frame can resist forces applied to its joints in any direction. The simplest space frame is a tetrahedron, and three additional members are required to fix each additional joint so that it cannot move in any direction. Hence the condition for simple stiffness of a space frame is

$$m = 3j - 6.$$

Remarks

Some structures are partly plane and partly space frames. For instance, the wing structure of a conventionally braced biplane is a space frame when considered as a whole, but the upper and lower wings separately are plane structures. There are usually more bays of drag bracing than of lift bracing, so that some of the drag bracing joints have no supporting members in the third dimension, loads normal to the wing being transmitted to the lift joints by the spars in bending. In view of such cases, care must be exercised in the application of these formulæ.

Redundant Structures

In a redundant framework the question as to which actual members are to be considered the redundant ones is often a matter for arbitrary choice. Alternative members may often be found, any of which may be removed without making any part of the structure incomplete.

As a redundant structure offers more than one path to the loads for a given system of external loading, the distribution of the loads in the members depends upon their relative stiffness and can only be determined by applying the principles of elasticity.

Redundancy due to Wire Cross-bracing

Where a panel is braced by cross-wires instead of by a single diagonal member capable of taking compressive as well as tensile loads, no extra redundancy is introduced provided the wires have no initial tension, as only one wire can work at a time, the other going slack.

If there is initial tension both wires are operative and there is, therefore, an added redundancy. This redundancy is present unless the loading is increased to such a value that one of the wires goes slack, in which case this redundancy is then eliminated (assuming this state of affairs to be reached before the structure fails). However, both the deflections and the load distribution in the redundant structure as a whole will not be the same as if there had been no initial tension, owing to the increased stiffness of the cross-braced panel before the load reached a value sufficient to make one wire go slack.

It is, perhaps, not always easy to see at first sight how a wire which is no longer operative at full load can influence the deflections and the load distribution in the other members. If this should be the case, it may be of help to realise that in the structure, when unloaded, if the wire which goes slack under load is removed, the opposite wire will also go slack and produce a " negative " deflection in doing so. Starting from this " deflected " position and applying the full loading, the final deflected position will obviously differ from that which would occur if there were no initial " negative " deflection and only the same single wire present. If the remainder of the structure is redundant, the distribution of the loads will obviously be affected by the change in deflections.

Weak Redundant Struts

A case somewhat similar to that of the cross-wire which goes slack is that of the weak redundant strut which cripples before the structure is fully loaded. In this case as the loading on the structure is increased, the strut continues to exert a small and almost constant force after crippling, though much less than just before crippling, so that the redundancy is partially, but not completely eliminated.

When such struts are very slender the " residual strength " is more or less negligible. The cross-bracing wire is an example of this.

It must be realised that this question of crippling is independent of, and extra to, the question of initial tension. As was seen above, the final loads are affected by initial tensions, even if one wire has slackened, but in the crippled strut case, if there was no initial load and the residual strength is negligible, the final loads are unaffected by the existence of the member.

Hooke's Law and the Principle of Superposition

The strain-energy theorems to be given are based on the assumption that Hooke's Law holds, so that the deflection of a point of application of a load is proportional to the load applied. Thus, if the structure is a pin-jointed frame, not only the actual material but each of its complete members must obey this law.

When the law holds, the principle of superposition applies. That is to say, the resultant effect of several systems of loading is equal to the algebraic sum of the effects of the systems considered separately.

The word " effect " used here applies to the loads in the members and the deflections. It does not apply to the strain-energy of the members since, for each member, this is proportional to the square of the load in that member, and " the sum of the squares is not equal to the square of the sum."

In spite of this it will be seen later on that the effects of more than one system of loading on a redundant structure can be added.

A Strain-Energy Axiom

An axiom on which strain energy methods depend is that the increase in the strain-energy of a structure due to applied loads is equal to the external work done on the structure by those loads.

The strain-energy of a structure may be expressed as the summation of the strain-energy of each of its members. Accordingly, expressions for the strain-energy of members subjected to various types of loading will first be given. These will then be followed by certain strain-energy theorems.

The Strain-Energy of a Member

The energy stored in a member when subjected to loading which gives rise to elastic distortion may be expressed in terms of the applied loading, constants depending on the shape of the member, and the modulus of elasticity of the material of the member. Such expressions, which are usually required in the application of strain-energy methods, may be obtained as follows for the various types of loading.

(i) DIRECT LOAD.

Let T = load in the member.
A = cross-sectional area of the member
l = length of the member
E = Young's Modulus of the material, and
U_m = strain-energy of the member.

When a direct load is gradually applied to a member of constant (EA) so that the load increases uniformly from zero to its final value, T, the average force applied during the process of loading is $\frac{1}{2}T$. At the same time the length of the member changes uniformly by a total amount Tl/EA. The work done on the member, being the product of the average load and the total change of length, may be expressed as

$$U_m = \tfrac{1}{2}T(Tl/EA) = T^2l/2EA.$$

When any or all of the quantities T, E and A are varying along the length of the member, the work done on an element of the member, of length δl, is clearly $T^2\delta l/2EA$ where T, E and A have then appropriate values for the particular element considered. Making δl indefinitely small, the strain-energy of the whole member is then obtainable by integration :—

$$U_m = \tfrac{1}{2}\int_0^l \left(\frac{T^2dl}{EA}\right).$$

(ii) BENDING (See Fig. 1).

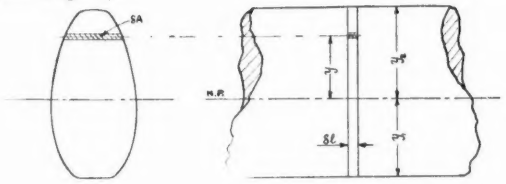

FIG. 1.

Let M = bending moment at any section along the member.
I = second moment of area of the section.
y = distance from neutral plane of any parallel layer.
δA = element of cross-sectional area at distance y from $N.P.$
δl = element of length of member at the section considered.

The load on δA = local stress $\times \delta A = (My/I)\delta A$. Then the extension of δl at y from $N.P. = \{ (My/I)\delta A \} (\delta l/E\delta A) = My\delta l/EI$.

Work done on elemental layer = $\frac{1}{2}$(load times extension)

$$= M^2y^2\delta A\delta l/2EI^2$$

,, ,, all layers in length $\delta l = \delta U_m = \sum_{y_1}^{y_2}\left(\frac{M^2y^2\delta A\delta l}{2EI^2}\right)$

$$= \left(\frac{M^2\delta l}{2EI^2}\right)\sum_{y_1}^{y_2}(y^2\delta A)$$

$$= \left(\frac{M^2\delta l}{2EI}\right)$$

Hence, strain-energy of whole member = $U_m = \frac{1}{2}\int_0^l \left(\frac{M^2dl}{EI}\right)$

Alternatively, this could be obtained by finding the product of the average *B.M.* applied to the section as it is loaded, and the angle through which the element of length is bent.

(iii) Torsion.

(*a*) *Member of constant section.*

Consider a member of length l and of constant section to which a twisting moment is applied gradually, starting from zero and reaching a final value Q. The angle of twist produced is at all times proportional to the applied moment, and its final value is θ. The total work done on the member will be the product of the mean twisting moment and the total angle of twist. Hence, the strain-energy of the member

$$U_m = Q/2 \times \theta$$

If K = torsion constant for the section, and
G = shear modulus of the material
Then $\theta = Ql/KG$
So that $U_m = Q^2l/2KG$.

If the member is of circular section, K becomes J, the polar second moment of area of the section.

(*b*) Member of Circular Section but Variable Diameter.

If the diameter of a circular member varies along its length,

$$U_m = \frac{1}{2}\int_0^l \left(\frac{Q^2dl}{JG}\right)$$

assuming the distribution of shear stress over the sections to be unaffected by the change of section. This assumption is to some extent equivalent to that made in the case of a member of varying section subjected to direct loading, the expression for the change of length Tl/EA being based on the assumption that the stress distribution is uniform over all sections.

In most practical cases the cross-section, if it varies at all, changes comparatively suddenly at a few sections. Hence the distribution of the stress departs appreciably from the assumed distribution for only a fraction of the length of the member, and the error involved in the use of the above expression for U_m is correspondingly small.

If the variation of section is gradual along the whole length it will usually be a comparatively slow change, so the distribution of stress will only be slightly affected, and again the error will be small.

(c) MEMBER OF VARIABLE NON-CIRCULAR SECTION.

In the case of a non-circular torsion member whose section varies along its length, a strict general expression for its strain-energy is not available, as the problem is complicated by the longitudinal strains which are set up. A solution of the problem is obtainable in one or two simple cases, such as that of an elliptic section varying uniformly along the member.

However, the error involved if the expression $\frac{1}{2}\int_0^l \left(\frac{Q^2 dl}{KG}\right)$ is used will not be very important if there are only one or two sudden changes of section in a reasonably "long" member, or if the change of section is gradual and comparatively slight. At the same time the approximation will probably not be so good as in a similar member of circular section.

The strain-energy of any member in torsion can be expressed in terms of the ratio twist/torque, when the latter can be determined experimentally.

(iv) SHEAR.

As the distribution of shear stress across a section varies with the shape of the section, no "ready-for-use" general expression for the shear strain-energy of a member of any section is obtainable. In practice it is sometimes necessary to consider the effect of shear, however, and an expression for the strain-energy may then be obtained as follows.

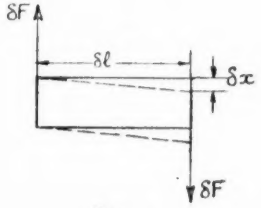

FIG. 2.

Consider an elemental block (Fig. 2) of length δl and cross-sectional area δA under a shear force δF, the shear deflection being δx. The shear stress across this element may be considered uniform.

Then, if δF is applied gradually, the work done is $\frac{1}{2}\delta F.\delta x$.

Now $\delta x = \delta l.\delta F/G\delta A$, where G is the shear modulus.

$\therefore$ Strain-energy of element $= (\delta F)^2 \delta l/2G.\delta A$

or $= q^2 \delta A.\delta l/2G$, where q = local shear stress.

Considering a complete section of a member, suppose it to be made up of a number of elemental layers parallel to the neutral plane, of width equal to that of the section at each layer, and of length δl parallel to the axis of the member (Fig. 1). Assume the shear stress is uniform across the width of each layer.

Then the strain energy of this complete element of the member is

$$\delta U_m = \left(\frac{\delta l}{2}\right) \int_{y_1}^{y_2} (q^2/G)\delta A$$

Hence the strain energy of the complete member is

$$U_m = \tfrac{1}{2} \int_0^l \int_{y_1}^{y_2} (q^2/G)\,\delta A\, dl$$

Expressions for q and dA, which depend upon the shape of the section, may be substituted in the above for any given case.

(Any error introduced by treating the shear stress as uniform across the width of the section will be very small in most practical cases.)

This expression for U_m may conveniently be reduced to definite forms for the simpler standard sections such as the rectangle and circle. By way of example, the case of the solid, rectangle of depth D and breadth B is given here. (The shear, F, is assumed to act in the direction of the depth.)

EXAMPLE.—Shear of solid rectangular section.

In this case the expressions for q and dA are as follows:

$$q=\left(\frac{F}{IB}\right)\int_{y}^{D/2} By\,dy=\left(\frac{F}{2I}\right)\left(\frac{D^2}{4}-y^2\right)$$

where I = second moment of area of the section.

$$dA=Bdy.$$

Then

$$\int_{y_1}^{y_2} q^2 dA=\left(\frac{F^2}{4I^2}\right)\int_{-D/2}^{+D/2} B\left(\frac{D^4}{16}-\frac{D^2y^2}{2}+y^4\right)dy$$

$$=\left(\frac{F^2B}{4I^2}\right)\left[\frac{D^4y}{16}-\frac{D^2y^3}{6}+\frac{y^5}{5}\right]_{-D/2}^{+D/2}$$

$$=\frac{F^2BD^5}{120\,I^2}$$

$$=\frac{6F^2}{5A} \qquad \text{since } I=\left(\frac{BD^3}{12}\right) \text{and } A=BD$$

Hence

$$U_m=\tfrac{1}{2}\int_{o}^{l}\left(\frac{6F^2}{5GA}\right)dl$$

$$=\left(\frac{3}{5GA}\right)\int_{o}^{l} F^2 dl$$

if GA is constant along the length of the member.

Further, if F^2/GA is constant along l,

$$U_m=\frac{3F^2l}{5GA}$$

The cantilever of constant rectangular section carrying a concentrated load at its end is the most common case covered by this last formula.

Had q been assumed uniform over the section, the coefficients in the above expressions for U_m for a rectangle would have been .5 instead of .6.

Remarks

(1) It will be seen later that it is the differential coefficient of the strain-energy with respect to some load or moment which is usually required rather than the strain-energy itself. In such cases it is worth noticing that it is not necessary to evaluate the strain-energy and then differentiate, as the above expressions may be differentiated before being applied. Thus, in the direct load case,

$$U_m=\tfrac{1}{2}\int_{o}^{l}(T^2/EA)\,dl$$

If this is to be differentiated with respect to a load P, then

$$\frac{\partial U_m}{\partial P} = \tfrac{1}{2}\int_o^l (\partial/\partial T)\ (T^2/EA)\ (\partial T/\partial P)dl$$

$$= \int_o^l (T/EA)\ (\partial T/\partial P)dl$$

Hence it is only necessary to multiply the expression for T by its differential coefficient instead of squaring the expression for T, integrating and then differentiating.

The expression may be simplified for certain special cases, thus:—

If EA is constant along the length of the member

$$\frac{\partial U_m}{\partial P} = \left(\frac{1}{EA}\right)\int_o^l T\left(\frac{\partial T}{\partial P}\right)dl$$

If T is also constant along the length of the member

$$\frac{\partial U_m}{\partial P} = \left(\frac{l}{EA}\right) T\ \left(\frac{\partial T}{\partial P}\right)$$

Further, if in this latter case $T = \alpha + \beta P + \Sigma\lambda Q$, where α, β and λ are constants,

$$\frac{\partial U_m}{\partial P} = \left(\frac{l}{EA}\right)(\alpha + \beta P + \Sigma\gamma Q)\beta$$

This form is convenient for use in " Least Work " calculations on redundant framed structures.

Similar expressions for the bending, torsion and shear cases corresponding to the above expressions for direct load are readily obtainable.

(2) All the above expressions for the strain-energy of a member and its differential coefficient are based on the assumption that the shape of the section does not distort under load and, in the case of members in direct compression, that bowing does not occur.

(3) It is not necessary to devote more space to the question of the strain-energy of members at present, as in most practical problems only the direct loads in the members need be considered in " Least Work " calculations. Sometimes bending problems have to be dealt with, but shear and torsion problems rarely arise.

This is due to the fact that in general the most economical structure for a given purpose is one in which the various parts are subjected to direct loads rather than bending or torsion. Hence the extensive use of framed-structures, the members of which are more or less pin-jointed. Even if the joints of such a structure are stiff the direct loads in the members are not usually greatly affected, so such loads are considered " primary " loads. Owing to the members being relatively stiff against direct loads as compared with bending, the small deformations of the framework due to direct loads generally give rise to very small bending effects. Offsets of the members at the joints may give rise to very important bending moments which may be aggravated by the end loads in the members which resist them, but even these offset moments may usually be regarded as " secondary," as they do not greatly affect the values of the direct loads. The structures may, therefore, usually be treated as pin-jointed for the purpose of determining the primary loads, and, if the framework is then redundant, the method of " Least Work " may be employed.

The secondary effects may, of course, be very important locally, and in the design of individual members will often exceed in importance the effect of the primary loads with which they are combined.

A few of the important strain-energy theorems will now be considered.

The Deflection of a Point in a Structure as a Function of the Strain-Energy.

Consider the application of a number of loads to a structure. Call any one of the loads P, and the remaining loads, which may be of differing magnitudes and acting in various directions, Q.

Apply all these loads gradually and simultaneously, starting from zero and reaching their final values together. For this given system of loading the deflections will then be proportional to the loads as the latter are increased. Then the work done on the structure by each load is the product of the mean load (*i.e.*, half the final value) and the final deflection, x, of its point of application in the direction of the load. Then the total strain-energy, U, of the structure is given by

$$U = U_0 + \tfrac{1}{2}Px + \Sigma\tfrac{1}{2}Qx \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where U_0 is the initial strain-energy, if the structure is self-strained.

If P is now increased by δP, all the forces Q remaining constant, each deflection x will increase by some amount, δx, which will generally be different for each load. Then the extra work done on the structure is

$$\delta U = P\delta x + \Sigma Q\delta x \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Alternatively, by differentiating equation (1) with P variable and Q constant, we have

$$\delta U = \tfrac{1}{2}P\delta x + \tfrac{1}{2}x\delta P + \tfrac{1}{2}\Sigma Q\delta x \quad . \quad . \quad . \quad . \quad (3)$$

Equating (2) and (3)

$$\tfrac{1}{2}P\delta x + \tfrac{1}{2}\Sigma Q\delta x = \tfrac{1}{2}x\delta P$$

Substituting in (2)

$$\delta U = x\delta P$$

Proceeding to the limit

$$\frac{\partial U}{\partial P} = x.$$

Remarks

The reason for the application being gradual is that the *static* deflection of the structure is being considered. If the loads were applied at their full value suddenly, the extra energy that would be imparted would be wasted in the damping out of the resulting oscillations of the structure.

The other restriction, *viz.*, that the loads must all start from zero simultaneously and increase uniformly at rates such that they all reach their final values simultaneously, enables the total work done by the loads to be expressed in the conveniently simple form $\frac{1}{2}\Sigma$ (load times deflection), for the deflections of all the points will increase uniformly and maintain constant mutual ratios (assuming Hooke's Law to hold and all the members to remain operative).

It will be noticed that the formula under consideration applies to any structure which obeys Hooke's Law since it is arrived at by considering only the external work done on the structure and any initial strain-energy due to self-straining is seen to have no effect on the final expression. The structure may therefore be simply stiff or redundant and, if the latter, it may be self-strained or not. Further, it does not matter whether the members are subjected to direct, bending, torsional, or shear loads provided they still obey Hooke's Law.

It should be mentioned that it can also be shown that, if M is a couple applied to a structure and θ is the resulting angular displacement of the " arm " of the couple about its axis, $\partial U/\partial M = \theta$.

It will be observed that it was not essential to separate the Q-loads and the P-load in the above proof; this was done in order to make clear the physical significance of the reasoning.

The Distribution of the Loads in a Non-Self-strained Redundant Structure (the Theorem of "Least Work")

Consider first the case of a redundant, pin-jointed framework.

Let P be the load in any one redundant member

Q be a general term for the various loads in the other redundant members

l, A and E be general terms for the lengths, cross-sectional areas and Young's Moduli of all the redundant members (A and E being assumed uniform for each member)

U be the strain-energy of the complete frame, and

U_1 the strain-energy of all the non-redundant members of the frame.

If the redundant member with load P is removed and replaced by equal and opposite forces P, with the same line of action, at its points of attachment, the loads in all the members will remain unaltered and the points of application of P will be in the same positions as if the member itself were still in place.

The strain-energy of the frame (excluding that of the removed member) will be

$$U_1+\Sigma\frac{Q^2l}{2EA}*$$

In the complete frame, if the points of attachment of the "P-member" have approached one another this member must be in compression; if they have separated it must be in tension. The relative deflection of these points is therefore negative (*i.e.*, in the opposite direction to the forces applied in place of the member) and its magnitude is given by the expression for deflection obtained above. Hence the amount of approach (or separation) of these points is

$$-\left(\frac{\delta}{\delta P}\right)\left[U_1+\tfrac{1}{2}\Sigma\left(\frac{Q^2l}{EA}\right)\right]$$

At the same time in the complete frame the P-member itself will have shortened (or lengthened) by an amount Pl/EA which must be the same as the relative deflection of its points of attachment.

$$\therefore\frac{Pl}{EA}=-\left(\frac{\delta}{\delta P}\right)\left[U_1+\tfrac{1}{2}\Sigma\left(\frac{Q^2l}{EA}\right)\right]$$

or, since

$$\frac{Pl}{EA}=\left(\frac{\delta}{\delta P}\right)\left(\frac{P^2l}{2EA}\right),$$

$$\left(\frac{\delta}{\delta P}\right)\left[U_1+\tfrac{1}{2}\Sigma\left(\frac{Q^2l}{EA}\right)+\frac{P^2l}{2EA}\right]=O$$

Now the expression in brackets is the total strain-energy of the complete frame.

$$\therefore\frac{\delta U}{\delta P}=O$$

In the mathematical sense this indicates that the load in any redundant member will be such that the strain-energy of the complete structure (which equals the work done) is a minimum or a maximum, but in the physical sense it can only be a minimum.

This is therefore called the "Theorem of Minimum Strain-Energy" or the "Theorem of Least Work." By its means the distribution of the loads in the members of a redundant structure can be found without any real difficulty, though in all but simple cases the amount of arithmetic involved may be somewhat considerable.

REMARKS.

(1) The strain-energy of each member of a structure must be considered positive, since it is proportional to the square of the load, whilst the length, area

* Assuming members of constant EA. As is shown later, this restriction is not essential.

and Young's Modulus are positive. The strain-energy of a structure may therefore be represented by a parabola (when plotted against a P-abscissa) which lies wholly " above " the P-axis. Thus it is obvious that in the physical sense $\frac{\partial U}{\partial P}=O$ can only refer to a minimum.

(2) In the proof as given above the case considered was that of a pin-jointed framework in which the value of the product of E and A was constant along the length of each individual member. This is a case frequently met with in practice and the proof is easy to follow since the physical significance of each step may be visualised readily.

The theorem is general, however, and not restricted to this case. A more general proof can be obtained as follows :—

Whether the members are of constant EA or not, the relative deflection of the points of attachment of a redundant member is given by

$$-\left(\frac{\partial}{\partial P}\right)(U_1+U_2)$$

where U_2=the strain-energy of the " Q-members," and the change in length of the P-member is

$$\frac{\partial U_3}{\partial P}$$

where U_3=the strain-energy of the P-member.

Hence, following the same line of argument as previously,

$$\left(\frac{\partial}{\partial P}\right)(U_1+U_2+U_3)=O$$

or

$$\frac{\partial U}{\partial P}=O$$

Further, by parallel reasoning, it can be shown that

$$\frac{\partial U}{\partial M}=O$$

where M is a redundant moment.

The former equation covers the case of redundant direct or shear loads, and the latter covers redundant bending or torsional moments, so that the theorem is seen to be quite general. Any of the so-called members, redundant or otherwise, may be parts of a structure which resist the force between their points of attachment to the rest of the structure by direct, bending, shear or torsional stress, and these parts may themselves be built-up, simply-stiff or redundant structures.

(3) It will be observed that it is not essential to separate the Q-members from the remainder of the structure in the proof of the theorem. It was done in order to make it clear that no effect of redundancies other than the one under consideration was being overlooked.

(4) In the form in which the proof of this theorem was first presented to the writer it was stated that the deflection of *each* of the two points of application of the P-loads was $-\left(\frac{\partial}{\partial P}\right)(U_1+U_2)$, in terms of the symbols used above, and that the relative deflection of the two points was therefore $-2\left(\frac{\partial}{\partial P}\right)(U_1+U_2)$. This seemed quite a reasonable application of the formula $\frac{\partial U}{\partial P}=x$ but, in order to get the right answer, it had, of course, been necessary to lose one of the $\frac{\partial U}{\partial P}$'s in the subsequent part of the proof. As others will have met the proof in the

same erroneous form, a word of explanation may be of use. The point is that the value of the deflection given by $\frac{\partial U}{\partial P} = x$ is the deflection of the point of application of P (in the direction of P) relative to the point or points at which P is reacted. Hence, in the case in question, one of the two equal and opposite P-loads may be considered as reacting the other, when the correct expression for the relative deflection of the two points follows immediately.

(5) It will be noticed that the Method of " Least Work " and the Method of Deformations sometimes used for solving redundant structures are ultimately similar in that they are both based on the equating of deflections. In the case of the former method this equating leads to the expression for " Least Work," whereas in the latter the equating is performed during each actual application of the method. Incidentally, as Southwell has shown, the principle of " Least Work " can be proved without resorting to this device.

(6) The loads in the members of a simply-stiff structure due to more than one system of applied loads may be found by adding algebraically the loads found for each of the systems considered separately.

Consider a framed structure with a single redundant member. Let the load in the latter be P. Then the load in each member can be expressed in the form $(\alpha + \beta P)$.

First apply one system of loading, then another separately instead of the first, then both simultaneously. In each case find the value of P. Let suffix 1 apply to System (1), suffix 2 to System (2) and suffix 3 to Systems (1) and (2) simultaneously. Then we have :—

System (1).

General term for load in each member is $(\alpha_1 + \beta P)$. Total strain-energy of structure $= U_1 = \Sigma \frac{l}{2EA}(\alpha_1^2 + 2\alpha_1\beta P + \beta^2 P^2)$.

Then

$$\frac{\partial U_1}{\partial P} = \Sigma\left(\frac{l}{EA}\right)(\alpha_1\beta + \beta^2 P).$$

By the theorem of Least Work, this equals zero. Hence

$$P = -\frac{\Sigma\left[\left(\frac{l}{EA}\right)\alpha_1\beta\right]}{\Sigma\left[\left(\frac{l}{EA}\right)\beta^2\right]} = P_1 \text{ (say)}.$$

System (2).

Load in each member $= \alpha_2 + \beta P$.

(Note that since the same member is considered to be redundant in each case, β is the same for each system, being dependent on the geometry only).

Then, as above

$$P = -\frac{\Sigma\left[\left(\frac{l}{EA}\right)\alpha_2\beta\right]}{\Sigma\left[\left(\frac{l}{EA}\right)\beta^2\right]} = P_2 \text{ (say)}.$$

System (3).

Applied loading = System (1) + System (2), simultaneously.

Then the load in each member $=\alpha_1+\alpha_2+\beta P$.

$$U_3=\Sigma\left(\frac{l}{2EA}\right)(\alpha_1^2+\alpha_2^2+\beta^2P^2+2\alpha_1\alpha_2+2\alpha_1\beta P+2\alpha_2\beta P).$$

$$\frac{\partial U_3}{\partial P}=\Sigma\left(\frac{l}{EA}\right)(\beta^2P+\alpha_1\beta+\alpha_2\beta)=O.$$

$$\text{Hence } P=-\frac{\Sigma\left\{\left(\frac{l}{EA}\right)(\alpha_1\beta+\alpha_2\beta)\right\}}{\Sigma\left[\left(\frac{l}{EA}\right)\beta^2\right]}$$

which equals P_1+P_2 although $U_3=U_1+U_2$.

Hence the principle of superposition applies to the loads in the members of a redundant structure and it is not essential that the " Least Work " calculations be performed *after* superposition. (When loads are to be superposed a strain-energy method should be used for each system of loading, unless it is known that $P=O$ in a given case).

The expression for $\frac{\partial U}{\partial P}$ is linear in terms of P, as is to be expected from the fact that $\frac{\partial U_m}{\partial P}=\frac{\partial U_m}{\partial T}\times$ constant (where $U_m=$ strain-energy of any member and $T=$ load in the member) and $\frac{\partial U_m}{\partial P}$ is the extension of the member.

It is easy to see now that the correct conclusion on this point is consistent with the general idea underlying the theorem of Least Work (and which is consequent upon Hooke's Law) that all deflections are proportional to the load whether the structure is redundant or not, assuming all the members remain operative.

The Distribution of the Loads in a Self-strained Redundant Structure

The symbols to be used will be the same as those used in the Least Work proof. The following are re-stated here for convenience :—

U = strain-energy of complete structure.
U_1 = strain-energy of structure less that of all redundant members.
U_2 = strain-energy of all redundant members except one (the " P-member ").
U_3 = strain-energy of the " P " redundant member.

Suppose the unstrained P-member is too short by an amount λ to fit in its place in the structure when all the other redundant members are in place (whether they produce self-straining or not), but no external loads nor the P-loads are applied. Consider the case in which the P-member is in tension, and call tension loads positive.

When the external and P-loads are applied the points of application of the P-loads will approach by an amount of

$$\frac{\partial}{\partial P}(U_1+U_2)$$

so that their distance apart will exceed the unstrained length of the P-member by

$$\lambda-\frac{\partial}{\partial P}(U_1+U_2)$$

Now since their final position under these conditions must be the same as

if the P-member were in place and the structure complete, this member must have lengthened by the same amount.

$$\text{Hence, } \lambda - \left(\frac{\partial}{\partial P}\right)(U_1 + U_2) = \frac{PL}{EA} \text{ or } \frac{\partial U_3}{\partial P}$$

$$\text{or } \frac{\partial U}{\partial P} = \lambda$$

Remarks

(1) In the above tension-member case, if the effect of applying the external loads is to increase the tension in the P-member above its initial tension load, the points of attachment of the member must actually move apart when the loads are applied. The numerical value of $\left(\frac{\partial}{\partial P}\right)(U_1 + U_2)$ will be negative since the relative deflection of the points of application is opposed to the applied P-loads. Thus the stretch of the P-member will be the arithmetic sum of the terms

$$\left(\frac{\partial}{\partial P}\right)(U_1 + U_2) \text{ and } \lambda.$$

(2) An additional term, U_0 representing the initial strain-energy of the structure (without the P-member) due to self-straining arising from other redundant members might have been included, but since this term would not vary with P it would have had no effect on the result. This has been shown to be the case in the proof of the equation $\frac{\partial U}{\partial P} = x$ so its inclusion in the above proof was considered unnecessary.

(3) Remark (3) on the Theorem of Least Work applies to this theorem also.

(4) It will be noticed that this theorem is not a theorem of least work, for the above equation shows that the strain-energy is only a minimum with respect to P when $\lambda = O$ (i.e., when the structure is not self-strained by the P-member).

(5) Observe that the P-load is unaffected by self-straining of parts of the structure other than the panel containing the P-member. This was implied in Remark (2) above and is accounted for in the physical sense as follows. The proportions of the load taken by the various paths in a redundant structure depend upon the relative stiffnesses of those paths. Provided no members such as initially tensioned wires or other slender struts cease to function under the imposed loads, the *stiffness* of a path is unaffected by initial tensions and compressions.

(6) As the above treatment of self-strained structures is frequently awkward to apply it is more usual to neglect initial loads in the first place, stress the structure for the applied external loads by " Least Work," and finally add the respective initial loads to the loads in the members thus determined, *provided all the members remain fully operative.*

The following proof is included to show that this method is valid for this condition.

Let U = total strain-energy of structure.
U_0 = initial ,, ,, ,, ,, due to self-straining.
U' = added ,, ,, ,, ,, ,, ,, external loading.
P_0 = initial load in P-member.
P_1 = added ,, ,, ,, ,, due to external loading.

It has been shown above that $\frac{\partial U}{\partial P} = \lambda$

Now $U = U_0 + U^1$

$$\therefore \lambda = \frac{\partial U_0}{\partial P} + \frac{\partial U^1}{\partial P} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

When $U^1 = O$, $\lambda = \frac{\partial U_0}{\partial P}$ and is constant.

Substituting this in equation (1) for any value of U^1

$$\frac{\partial U_0}{\partial P} = \frac{\partial U_0}{\partial P} + \frac{\partial U^1}{\partial P}$$

$$\therefore \frac{\partial U^1}{\partial P} = O$$

Now $P = P_0 + P^1$

$$\therefore \frac{\partial P}{\partial P^1} = 1$$

Hence

$$\frac{\partial U^1}{\partial P} = \frac{\partial U^1}{\partial P^1} = O$$

That is to say, the strain-energy due to the external loading (neglecting that due to self-straining) is a minimum with respect to the increase in the load in the redundant member due to the application of the external loading.

(7) The conclusion reached in Remark (6) on the theorem of Least Work in regard to the principle of superposition cannot be extended to apply to this theorem for self-strained structures without reservation. The loads in the members for different loading systems found by putting $\frac{\partial U}{\partial P} = \lambda$ cannot be superposed directly. If λ is substituted for zero in each case in the proof given in the Remark referred to above it will be seen that superposition of the loads found for Systems (1) and (2) would take into account the effect of λ twice over.

If it is desired to superpose loads in the case of a self-strained structure, it is therefore necessary to either (*a*) solve for one loading system by putting $\frac{\partial U}{\partial P} = \lambda$ and all the others which are to be added by putting $\frac{\partial U}{\partial P} = O$ or (*b*) solve all the systems by Least Work and add the initial loads.

PART II.

Application

First of all it is as well to observe that the design procedures for simply-stiff and statically-indeterminate structures differ in the following respect.

The loads in the members of a simply-stiff structure under a given system of external loading may be determined immediately if the geometry of the structure (when each member is represented by its centre line only) is known. The members can then be designed round these centre lines to withstand the loads thus found.

In the case of a statically-indeterminate structure, however, the procedure is not so straightforward, being complicated by the fact that there is more than one path for the loads and that the distribution of the loads between the various members depends upon the relative stiffnesses of the different paths. The stiffnesses of the paths depend upon the cross-sections and effective moduli of elasticity of the members as well as upon the geometry of the structure, so it is necessary to know the shape, size and material of the members before their loads can be determined.

In designing a redundant structure it is therefore necessary first to guess suitable sizes for the members and then to determine the loads in them. If any of the guessed members prove unsuitable for these loads the members must be modified and the new loads found for these modified members. This process of trial must be repeated, if necessary, until all the members are satisfactory.

The designer can usually contrive to make fairly lucky " guesses " on the whole by considering the loads that would be experienced by the members if the redundant members were removed, together with the probable effect of the redundant members. In practice, therefore, this process of trial does not generally prove very troublesome.

General Method of Application of the Principle of Least Work

The expression for the strain-energy of the complete redundant structure is differentiated with respect to the loads in each of the n redundant members in turn. The differential coefficients are equated to zero or λ (as the case may be) giving n equations which, when solved simultaneously, determine the loads in the redundant members and hence in all the other members of the structure.

(Redundant moments may, of course, replace some or all of the n redundant loads in the above.)

Method of Application of the Principle of Least Work to a Non-Self-strained Redundant Framework

The above general statement of method will be made clearer by a slightly more detailed statement of method for this particular class of redundant structure. The method of application is then as follows :—

Remove the redundant members and replace them by forces, P_1, P_2, P_3, etc., applied at the points where they were attached. (Each member will be replaced by two equal and opposite forces acting in the direction in which the member lay.) This leaves a simply-stiff frame under the action of the forces P_1, P_2, etc., in addition to the original external forces. The loads in all the members can then be found in terms of the original external loads and P_1, P_2, etc., by the ordinary static methods.

The values of $\frac{\partial U_m}{\partial P_1}$, $\frac{\partial U_m}{\partial P_2}$, etc., can then be obtained for all the individual members and summed to give $\frac{\partial U}{\partial P_1}$, $\frac{\partial U}{\partial P_2}$, etc.

Each of these latter differential coefficients is equated to zero and the resulting equations solved simultaneously, giving the values of P_1, P_2, etc.

The loads in all the members, previously known only in terms of the " P-loads," can then be evaluated.

Remarks

For simple structures, especially plane frames, ordinary stress diagrams usually provide the quickest solution, though for frames of very simple geometry (such as the drag truss of a wing in which the spars are parallel and are braced by perpendiculars and diagonals) calculation is often even quicker.

In the case of space frames it is generally best to calculate the loads by considering the equilibrium of each joint in turn. This may be done by direct cosines or tension coefficients. The former give the loads direct but the direction cosines of the members must first be evaluated. The latter give first the values of the products of the load and length of each member, and thence the loads themselves. The difference is simply whether the components of the members shall be divided by the true length before solution of the equations or whether the division shall occur after solution of the equations.

If the structure has to be treated for only one system of loading there is probably nothing to choose between the two slightly different methods, but if the structure has to be treated for several different systems of loading the cosine method is a shade quicker as the division mentioned above does not have to be repeated for each system.

When there are not more than three members meeting at any joint, each joint can be solved completely in turn, but when more than three members meet at some or all of the joints it will often be necessary to solve simultaneously the equations for at least two of the joints.

A slide-rule is generally used for the calculations, as the use of logarithms giving a larger number of significant figures would involve too much time. This often leads to the accumulation of error as each successive joint is dealt with. For this reason it is advisable to calculate the correct final reactions directly from the applied loads and overall geometry and compare with those given by the loads which have been found in the members meeting at the reaction points. It is always wise to check the arithmetic, as each joint is solved, by substituting the solutions back into the equations. Small errors can usually be eliminated sufficiently well by suitable adjustment.

It is usually simplest to find the loads in all the members due to the external forces and to the loads in the redundant members separately. (These loads may be termed the " constant loads " and the " P-loads " respectively). They may then be superposed and the Least Work calculations carried out, preferably in tabular form. These calculations are a matter of arithmetical labour only and not of mathematical difficulty. In fact, they may be done mechanically without any understanding of the underlying principle.

Hooke's Law and Effective Stiffness of Members

The deflection equation $\frac{\partial U}{\partial P}=x$ is true only when $Pdx=xdP$, that is to say, when x is proportional to P. Since the Theorem of Least Work can be proved by the use of this equation it follows that Hooke's Law must be valid if the Theorem is to hold.

Most of the structural materials employed obey the law very closely at low stresses and fairly closely nearly up to the " yield point."

Some difficulty is encountered here, however, in defining the yield for, while one or two materials (such as mild steel) exhibit a clearly defined yield which is not over-sensitive to test conditions, most of the other commonly-used materials show no definite yield. Thus many high tensile steels give a smooth load-deflection curve which falls over more and more rapidly as the load is increased, while duralumin usually shows a series of yields which may vary in number and position on the curve. Further, the slopes of the curves for all materials vary with rate of loading and other test conditions.

When there is no definite yield for a material it is usual to substitute for it a certain " proof stress " (often the " 0.5 per cent. proof stress ") for design purposes. While it is true that by suitably enlarging the deflection scale it is possible to make the " true " elastic limit appear to have almost any value lower than the yield yet, in spite of all the variations, it is usually possible to fix on fairly satisfactory mean values of Young's Modulus for practical purposes.

The question then arises as to whether the actual maximum stresses will not exceed the " yield." This depends largely upon load factors and what they imply. The necessary load factors for design may be laid down but they have only a somewhat vague significance unless it is also definitely laid down what the condition of the structure must be when subjected to the factored loading. Obviously the structure should not " fail " before the loading reaches its full

value, but it is often very difficult to define an actual failing point as there are usually degrees of failure. Members and attachments usually suffer serious and permanent distortion long before actually breaking up.

There are two main possible alternatives for the allowable condition of the structure at the factored load; it may either be up to its ultimate load or it may be up to its yield. In the first case the structure will just not fall to pieces if the load is *just* below its fully factored value, but the structure will be badly deformed in all probability, while in the second case it will presumably be "as good as new" when the load has been removed, any permanent set being negligible. It is merely a question of the definition of the load factors which decides which alternative is to apply.

Further, owing to the difficulty of defining the yield point, in order fully to define the strength significance of a given load factor based upon yield, the "yield" stresses for the various materials for use in conjunction with the load factors must also be laid down.

There appears to be a definite tendency to limit the stresses to the yield at full factor. This may be due in part to the fact that higher accelerations have been recorded and are considered likely to be imposed generally, and in part to a desire for greater durability and length of working life.

A minor point which also affects the stresses actually occurring is that it is rarely possible to find a practical size of member which is only just strong enough for its design load, so that there is usually a larger factor of safety than called for, resulting in a lower stress than the allowable. This obviously cannot be relied upon for all the members of a structure.

With the above points in mind it is possible to consider the question of Hooke's Law in relation to the main classes of members commonly met with (viz., wires, struts and spars) in slightly more detail.

Wires

These are generally designed on their ultimate strength. Overlooking the fact that they will usually develop a strength in excess of the nominal figure, as they will probably exceed the specification minimum stresses and areas, it is clear that they will not obey Hooke's Law up to the fully factored load.

However, the load factors certainly do contain appreciable factors of safety for all accelerations likely to be imposed during reasonable flying manœuvres, even if the actual maximum loads are somewhat in excess of the nominal half of the "Centre of Pressure Forward," "Centre of Pressure Back" and Inverted Flight load factors. The wires should, therefore obey the law quite closely up to the working loads in these cases. As regards the Nose Dive case, the actual factor of safety is larger than would appear at first sight owing to the fact that the ceiling of the machine rarely permits of the terminal velocity being attained and further, that in normal flying no attempt would be made to approach it very closely though, against this, it must be remembered that an increase in tail load will be required to pull the machine out of the dive.

The factors of safety are certainly smaller in the landing cases since the imposed accelerations may often be unintentionally severe due to bad ground surfaces, however good the piloting may be. However, in the case of most conventional machines the structure need only be treated as redundant in possibly the side-load landing case, and usually most of the fuselage members which are heavily loaded by the undercarriage are struts rather than wires. The most likely case of a redundant wire-braced structure being subjected to serious landing-loads occurs in the multi-engined biplane.

With this latter possible exception, it would appear that wires as designed at present may be expected to obey Hooke's Law closely up to the maximum working loads, though not up to the fully factored loads.

Struts

As the mean allowable stress for a strut as given by a Southwell curve must be below the yield stress, and is usually considerably below it, it may be assumed that the material of strut members will obey the law with considerable accuracy.

Against this must be set the question of deflection owing to eccentricity of loading. If the strut is at all long it will bow considerably under its limiting load. The ends will therefore approach more than they would under pure compression, thus giving a reduced effective value of Young's Modulus for the member. If the strut is very long it will cease to obey Hooke's Law at all closely as the ends will approach more and more rapidly as the end load is increased. This effect will probably be small in most practical cases.

The position of struts in relation to Hooke's Law may therefore be considered satisfactory on the whole, even up to the fully factored loads, though the use of " effective E's " in place of the true E for the material may be desirable in some cases.

Spars

The departure from Hooke's Law will generally be most pronounced in the case of members, such as wing spars, under combined end load and lateral bending moment (at any rate up to maximum working loads).

This introduces the question of the work done by the end load bending moment. This could be obtained in general form from the Berry expression for the bending moment at any point in a bay by substituting into this expression the expression of " M_B " derived from the Berry Three Moments equation, and differentiating with respect to the load, Q,* in the redundant member. This would give $\frac{\partial M}{\partial Q}$, which would have to be multiplied by M and then integrated with respect to x for the whole bay.

A more promising line of attack is to plot the bending moment diagram for the spar for various values of Q. Corresponding curves of M^2 can be derived and the areas of these, representing $\int_0^l M^2 dx$, can be found. Hence a series of corresponding values of Q and U_m due to the total bending moment can be obtained.

The curve showing the relation between U_m and Q would be a parabola if Hooke's Law held, and its slope, which could be expressed in the form $a + bQ$ (where a and b are constants) would be the required value of $\frac{\partial U_m}{\partial Q}$ for inclusion in the " Least Work " summation.

Actually the U_m curve would depart from a parabola (and hence the $\frac{\partial U_m}{\partial Q}$ curve from a straight line) to a greater or less degree according to the conditions. The use of a mean straight line for $\frac{\partial U_m}{\partial Q}$ should give a reasonable approximation provided the spar deflections under full load are not excessive. It should certainly be better than neglecting the effect altogether.

The procedure would then be to find a first approximation to the value of Q by neglecting the work done by the end load bending moment. Find U_m as above for this value of Q and also for the value of Q which makes P, the spar

* Q has been used here, instead of the P previously used, in order to avoid confusion with P, the conventional Berry symbol for the spar end load.

end load, equal to zero. If the Berry effect is pronounced, values of U_m for other values of Q should be found and a mean $\frac{\partial U_m}{\partial Q}$ derived as mentioned above, but if the Berry effect is reasonably small it should be sufficient to find the slope of the parabola through the two first-mentioned points, the second of the two points being the minimum value of U_m.

Reverting to the case when the Berry effect is pronounced, if the inclusion in the " end load bending moment $\frac{\partial U_m}{\partial Q}$'s " should affect the value of Q appreciably it may be necessary to revise the values of the mean $\frac{\partial U_m}{\partial Q}$'s which were used, thus giving a third approximation. Such a case is hardly likely to occur in practice, however.

Another matter which requires investigation is the value of Young's Modulus, which should be used for thin metal members such as are used for spars. As the result of official tests, the value of E to be used for a given spar is laid down. This may differ appreciably from the normal value of E for the material itself.

It should be realised that the expressions for the strain-energy of a member given in the beginning of this paper are based on the assumption that the section does not change its shape under load. As the metal members used in aeroplanes are usually of thin material, their sections may distort appreciably without failure. Such distortion would generally be quite small in members subjected to direct loads, somewhat larger in those subjected to bending and shear, and probably of considerable magnitude in torsion cases (with the exception of circular members subjected to torsion). The effect of this distortion can usually be allowed for by the use of a modified value of E.

Details

The effect of details such as joints and attachments will be to modify the effective E of the members. The mean stresses in the details may possibly be lower than in the members but local stresses may be high with many systems of construction. The resulting effective E's cannot be calculated easily or accurately so that tests on the particular types of construction that are to be used would be necessary to determine them.

However, the details probably affect the values of the effective E's of the members to a very small extent in most cases, and it will usually be sufficient to consider the sections of the members as extending the full lengths between joint centres and neglect the details.

Offsets of members at the joints tend to increase the effective softness of the structure. Their effect, if important, can usually be included with sufficient accuracy by suitably modifying the values of E for the members affected by the offset moments.

Effect of Initial Tensions

Aeroplane structures are frequently self-strained owing to the use of cross-wire bracing which is initially tensioned.

When the initial tension in a pair of cross-wires is less than half the load that would occur in the single wire if there were no initial tension, one of the wires will go slack before the full load is reached; when it is more, they will both remain operative under the load.

In the former case, if the structure becomes simply-stiff when one wire slackens, the loads in the remaining wire and all the other members are the same as if there had been no initial tension. If, however, the structure remains redundant when

one wire slackens the condition of affairs is more complicated, for the rate of deflection of the panel with increase of load is different according to whether both or only one of the wires is working. The panel then no longer obeys Hooke's Law over the whole of the working range, so that the methods given at the beginning of the paper are not strictly applicable.

In the latter case (*i.e.* when both wires remain operative) the structure can be treated strictly by Least Work—with the effect of the extra redundancy due to the second wire included—and the initial loads added. However, assuming the pair of wires to lie in the same plane, the effect of the second wire can be allowed for approximately by adding its stiffness to that of the first wire. (This assumes that the struts completing the panel are stiff compared with the wires). The Least Work can then be performed as usual with the single virtual wire working. The load found for the virtual wire would have to be divided between the actual wires in proportion to their stiffness. The initial tension loads would then have to be added to the results.

If a strict solution of the problem is attempted by the application of the formula $\partial U/\partial P=\lambda$, an extra redundancy is introduced for every panel which has its second wire working, and the solution of the resulting simultaneous equations becomes too long a job for ordinary stress purposes.

There is also the difficulty of determining λ for given initial tensions. For a single cross-braced panel there is no great difficulty, especially if the softness of the struts is neglected, but when a complete redundant structure such as the wing structure of a biplane is considered, the problem assumes another aspect. The lift trusses are usually inclined to the drag trusses, and if the corresponding lift and anti-lift wires do not lie in the same plane the initial tensions in the lift truss wires will effect the initial tensions in the drag truss wires. It is easy enough to find the extension of any wire for its initial tension, but the remaining part of λ (*i.e.* the amount by which the points of attachment approach one another) can only be found in terms of the loads in the redundant members, which loads depend upon the respective values of λ itself! In other words, it is of no use trying to find the relative deflections of the points of attachment by means of the equation $\partial U_1/\partial P=x$ (where U_1 is the strain-energy of the structure without the P member under consideration) since the equation $\partial U/\partial P=\lambda$ is derived from it.

If each of a pair of cross-wires lies in the same plane, approximate values of λ can be determined fairly quickly by considering the stretch of the wires only. As mentioned before, if one of a pair of initially tensioned wires is cut, considering an independent pin-jointed panel, the panel will distort under the action of the remaining wire, the load in which will become zero. It can therefore be seen that the required approximate value of λ is equal to the sum of the extensions of the two wires under their initial tensions.

However, it is impossible to make any strict allowance for initial tensions in practice without ensuring by some means or other that certain specified initial tensions are introduced in all the wires when the machine is rigged. This is surely quite unnecessary and impracticable, and obviously the initial tensions could not be maintained constant while the machine was in use.

Fortunately in practice the initial tensions are comparatively small, and for any case in which that part of structure in which they occur carries much load, and is therefore of importance from both the structural strength and strain energy points of view, they are practically swamped out. No serious error is therefore to be expected if they are neglected in the usual way.

Load Factors for Redundant Biplane Wing Structures

Although the Load Factors were arrived at originally rather arbitrarily, they have since been placed on a more logical basis as the result of measurements of the accelerations which are actually imposed in flying, and the Load Factors

for at least the C.P.F., C.P.B., and I.F. cases may be considered to contain a factor of safety of something approaching two on the maximum flying loads.

This factor of safety should be sufficient in view of the comparative accuracy of the stressing, the high grade of materials and thoroughness of inspection employed in aeroplane work. Of course, it has to cover many unconsidered effects and minor uncertainties, such as loads from ailerons and unconsidered manœuvres, vibration stresses and fatigue effects, concealed defects in material and workmanship, initial tension loads and various neglected secondary stress effects, but there should not be any call for an additional 20 per cent. on various odd parts of the structure.

There remains the question of the reliability of the incidence bracing as an effective part of the structure. When this bracing takes the form of a strut, it must be assumed to be as reliable and effective a member as the other interplane struts. The Least Work loads should therefore be accepted for design purposes without any addition such as the 20 per cent. at present required.

When the incidence panel is braced by cross-wires, the Least Work calculations will be carried out for only one incidence wire working, initial tensions being neglected as usual. (If both wires worked, the effectiveness of the incidence panel would be increased, but the increase would be small in practice as argued above in connection with initial tensions generally.)

Now the incidence wire might fail, or at least be rigged slack so that it would not be fully effective, in which case the conditions attain or approach those of the no-incidence-bracing case. In order to cover these possibilities, it is suggested that "incidence-wire cut" cases be included in the stressing in addition to the cases solved by Least Work. The latter would be the normal or basic cases in place of the former as at present. These incidence-wire cut cases would be stressed for half the normal factor just as is done for cut lift wire cases. Very little extra work would arise from them, as the incidence-wire cut loads will have been obtained for the Least Work calculations, and they will only have to be halved owing to the reduced factor.

It is interesting to note that the replacement of an incidence strut by a wire giving only the required strength has a comparatively small effect on the Least Work loads. In one or two cases in which this substitution was tried, the wire load was of the order of 90 per cent. of the corresponding strut load. One case gave 94 per cent. and this did not appear to be exceptional. Physically, this is due to the stiffness of the incidence-panel for its loading as compared with the lift drag and centre section trusses for their loading, which arises partly from the "better angle" of the incidence wire, as compared with the lift wires, generally found in single bay biplanes. The point is that it is not reasonable to treat only biplanes with incidence *struts* by Least Work and not those with incidence *wires*.

Remarks

Before summing up the above, there are a few odd points that may be mentioned.

(1) In all Least Work problems it is advisable to include all the members and not only those expected to be important, such as wires, unless there is definite evidence that this is unnecessary for the given type of structure under the given type of loading. Sometimes experience will show that certain parts may be neglected for certain standard stressing cases. For instance, it may be found that the centre-section members, excepting the centre-section spars, have a comparatively negligible effect on the wing loads, or that for a certain flying case the incidence-bracing load is negligible, but let it be emphasised that this cannot be relied upon for slightly different structures and that the extra work involved in making sure is usually small compared with the total.

(2) It is worth noting that although the strain-energy methods are concerned with extensions and deflections of members and structures, yet it is assumed that the geometrical distortion is not sufficient to alter the relative angular attitude of the members of a frame structure, and hence the loads, appreciably. In most cases the effect is negligible up to the yield, though it may become appreciable near the ultimate load.

(3) Those whose confidence may be shaken by the uncertainties encountered in connection with aeroplane stressing may find reassurance in the fact that the effect of initial tensions need not be feared in all ordinary practical cases as one of the pair of wires will certainly have gone slack before the surrounding members have reached their failing point. The initial tensions will in general not be large compared with the failing loads and the residual effect should not be of much importance. Also, when one part of a redundant structure is over-loaded it " gives " slightly and thus allows the other parts of the structure to give further assistance.

(4) An effect which has been neglected is the shortening of the spars due to bending under lateral loading only, apart from that due to end load compression and bending. This is equivalent to initial tension, being independent of the redundant load.

Thus, if the upper front and rear spars are shortened by this bending the interplane strut joints rise, generally by different amounts, thus straining an incidence wire which would in turn produce loads in other members such as drag wires.

In conclusion, the author wants to express his thanks to Mr. H. F. Vessey, B.Sc., for his valuable assistance with the checking of the proofs.

COMPRESSIBILITY EFFECTS IN HIGH SPEED AIR FLOW

BY

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(*Paper read before the Students' Section.*)

Any theoretical attempt to evaluate the forces and couples experienced by an aircraft when in flight by a mathematical analysis of the pressures exerted by the air when in motion about the various parts, leads to what have, so far, proved insuperable difficulties. It involves the integration of the equations of motion of a real fluid, and, except in a few very special cases, these have been insoluble. The actual motion of a fluid is affected by a number of its properties, and, in general, accounts would have to be taken of its density, viscosity, and at high speeds its compressibility. In certain circumstances the effect of these last two can be neglected and the classical theory of hydrodynamics dealing with the motion of a non-viscous, incompressible or perfect fluid can be applied. A further simplification consists in supposing that the motion is irrotational, that is, any small portion of the fluid at a point has no angular velocity about its centre of gravity. If a further restriction is made that the particles all move in one plane, or that the motion is two dimensional, then it is possible to determine the flow of a perfect fluid in most cases.

For example, the irrotational motion about a circular cylinder can be quite easily determined. If there is no circulation of the fluid about the section, then the symmetry of the motion clearly indicates that there will be no force on the body, and calculations by means of the pressure equations verify this. If there is a circulation, then the symmetry of the streamlines about the direction of general streaming is destroyed and there will be a force acting at right angles to the direction of the stream at infinity.

This is a well-known property of a perfect fluid. Provided the motion is irrotational, in no circumstances can it exert a drag upon a body, and if a circulation exists then there will always be a lift force acting at right angles to the direction of general streaming.

The drawback in the application of the classical theory to the motion of a real fluid is now obvious. It is a well-known fact that bodies in motion do experience a drag; but the theory can be applied in some cases to give results which, apart from the difference in resistance, do give good agreement in other respects with actual observed facts. The most successful of these applications to aeronautics is due to Joukowski, who discovered that a circular cylinder could be transformed into a sharp-tailed section—such as an aerofoil—by a simple mathematical theorem. The flow pattern about the cylinder transforms at the same time, and, unless the circulation has a special value, then an infinite velocity at the sharp tail will be derived. This state of affairs is, of course, not realised in practice where viscous action prevents anything approaching this high velocity from occurring. The Joukowski hypothesis is that a circulation must be superposed upon the general streaming, and of such an amount that the streamlines leave the sharp tail smoothly. Under these circumstances, as in the case of the

cylinder, there will be a lift upon the aerofoil, and the calculated value of this lift agrees very closely with the observed force experienced by a similar aerofoil when placed in a wind tunnel. In the latter case a drag will be encountered, but at low speeds this is very small, and all the evidence points to the fact that the Joukowski flow with circulation is a good approximation to what actually occurs.

The preceding theory is based upon the assumptions that the fluid is incompressible and non-viscous. Experiments have shown that the effects of compressibility are small until velocities approaching that of sound are reached. For aircraft having a maximum speed of the order of 150 m.p.h. it is quite safe to neglect the effects of compressibility, but our fastest machine now travels at a speed in the neighbourhood of 360 m.p.h., or approximately half the velocity of sound, and at this high velocity the assumption that air is incompressible is no longer tenable.

Let us consider the fundamental differences between an elastic and an inelastic fluid. The velocity of sound in a medium is by definition the rate of propagation of small disturbances, and is given by

$$a^2 = \frac{E}{\rho}$$

where E is the elasticity and ρ is the density. In the case of a gas

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{dp}{(d\rho/\rho)}$$

and hence $a^2 = \dfrac{dp}{d\rho}$

For an incompressible fluid $d\rho = o$ and therefore a^2 is infinite, and the more compressible the fluid is, the smaller is the velocity of sound. For air at atmospheric conditions its value is approximately 1,100ft. per sec.

Consider now the flow of a compressible fluid along an elementary stream tube of small variable section S. If q is the fluid velocity, then the mass of fluid which flows across any section is constant, and the continuity equation takes the special form

$$\rho S q = \text{const.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Differentiate this with respect to q and

$$\left(\frac{1}{\rho}\right)\left(\frac{d\rho}{dq}\right) + \left(\frac{1}{S}\right)\left(\frac{dS}{dq}\right) + \frac{1}{q} = o. \quad . \quad . \quad . \quad . \quad (2)$$

The integral of the equations of motion of the fluid along the stream tube connects the pressure, density and velocity together and is known as Bernoulli's equation. It is

$$\int \frac{dp}{\rho} + \tfrac{1}{2}q^2 = \text{const.} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Differentiate this with respect to q.

$$\left(\frac{1}{\rho}\right)\left(\frac{dp}{dq}\right) + q = o. \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Equation (2) can be rewritten

$$\left(\frac{1}{\rho}\right)\left(\frac{d\rho}{dp}\right)\left(\frac{dp}{dq}\right) + \left(\frac{1}{S}\right)\left(\frac{dS}{dq}\right) + \frac{1}{q} = o.$$

and substituting in this for

$$\left(\frac{1}{\rho}\right)\left(\frac{dp}{dq}\right)$$

from (4) we have

$$\frac{dS}{dq} = -\left(\frac{S}{q}\right)\left[1 - q^2/(dp/d\rho)\right]$$

or

$$\frac{dS}{dq} = -\left(\frac{S}{q}\right)\left(1 - \frac{q^2}{a^2}\right). \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where a is the local velocity of sound.

From this equation it follows that if $q<a$ then dS/dq is essentially negative and S decreases as q increases. When $q=a$, $dS/dq=o$ and the section is a minimum. If $q>a$, dS/dq is positive and S increases as q increases. If q steadily increases along the tube from a value $q<a$ to $q>a$ then the tube converges to a minimum section when $q=a$ and then diverges as q exceeds a. If we make a infinite in equation (5) then we obtain the corresponding formula for an incompressible fluid, namely

$$\frac{dS}{dq} = -\frac{S}{q}.$$

Hence, in this case, dS/dq can never vanish, and the least section of the stream tube is reached at the position of maximum velocity along the tube.

The fact that the minimum section of a stream tube is reached when the velocity attains that of sound has an important bearing upon the motion of compressible fluids. It is evident that the flow pattern past a body will change considerably when the local velocity of sound is exceeded at any point; and we might expect this change to show itself in the forces which act upon the body.

The above theory has an application in the following interesting experiment. Air is supplied from a large boiler under pressure to a pipe which converges to a minimum section and then remains constant. By decreasing the pressure at the open end of this constant section the fluid can be speeded up until the velocity of sound is reached at the minimum section. After this any further decrease of pressure has no effect on the fluid in the pipe, and any increase of pressure in the boiler merely serves to increase the density of the air current while leaving its velocity unchanged. This phenomenon has an obvious explanation. The disturbance caused by decreasing the pressure at the open end will travel up stream with the velocity of sound and will simultaneously be swept down stream with the speed of the fluid. When these velocities are equal the nett effect is that the disturbance cannot reach the minimum section, and can therefore have no effect there.

In another series of experiments Sir Thomas Stanton has examined the velocity distribution in the neighbourhood of a throat in a circular pipe. The constriction consists of a circular sleeve whose longitudinal section is two circular arcs of large radius. The results are shown in Fig. 1. In diagrams A and B the flow is symmetrical about the throat, and it appears from the analysis of Professor Taylor that this state of affairs will continue until a velocity which is very close to the local velocity of sound is attained at the point of maximum velocity in the field. This point would correspond to a position in the central plane of the constriction very close to the wall. The curves C and D correspond to transition states in which the velocity of sound is exceeded in regions of the field, and finally in curves E and F a definite non-symmetrical flow has developed. In this type of flow the velocity increases steadily along every stream tube and there is no position of maximum velocity. Also only one such type of flow can exist, for it will be noticed that, although the initial pressure in the drums has been increased, the curves E and F remain substantially the same.

The property of air expansion after the velocity of sound has been reached is made use of in obtaining high velocities in the High Speed Wind Tunnel at the National Physical Laboratory. This tunnel was constructed and used by Sir Thomas Stanton to investigate the effects of compressibility upon models. A sketch is shown in Fig. 2. The air supply is obtained from a 530 h.p. air compressor and is delivered through a screw valve to a converging mouthpiece. It was found that the shape of this mouthpiece had an important influence upon the velocity distribution in the tunnel, and the best type was arrived at by experiment. For speeds below the velocity of sound a parallel nozzle was used, and with this the highest speed that could be attained in the working section was 0.85a. As the previous experiments indicated, any increase in the initial supply pressure merely increased the density of the air stream. By using a divergent

mouthpiece velocities in excess of that of sound could quite easily be obtained. The presence of the model, however, limited the range of velocities which could be used in any particular experiment. For velocities slightly greater than a, the constriction of the tunnel, due to the model, was greater than that caused by the diverging mouthpiece, and consequently the region of air expansion transferred itself to just behind the model. Due to this it was found to be impossible to use the range of velocities between approximately 0.8a. and 1.5a.

In the working section the model is rigidly held in a metal ring lying wholly in a recess of the wall of the tunnel. Such a recess has no effect upon the

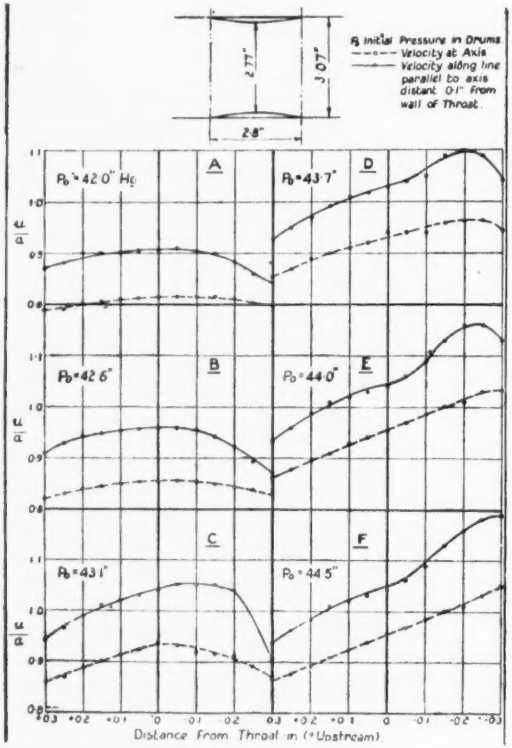

FIG. 1.

velocity distribution provided its dimensions are within certain limits. This ring is capable of support upon three alternate axes for determining the lift, pitch and moment. The general arrangement can be seen from the figure.

Sir Thomas Stanton determined the velocity distribution along and at right angles to the axis of the working section and found them to be remarkably smooth and steady. The tests which I will describe here were made by him upon two aerofoils

(1) a thin bi-convex circular arc section
(2) R.A.F. 31a.

and are given in detail in R. and M. 1130. The lift coefficients and the slope of the lift curves increased for R.A.F. 31a. as the tunnel speed increased up to approximately 0.6a, the drag meanwhile remaining low. The evidence points

to the fact that irrotational flow with a modified Joukowski circulation is a good representation of the actual motion. At speeds above 0.6a. the lift coefficients rapidly decreased and the resistance increased to a very large value. The Joukowski circulation formula no longer gave the lift experienced and the ordinary theory failed to represent the motion adequately. One rational inference appears to be that at a value of approximately 0.6a. irrotational motion breaks down and ceases to be possible about R.A.F. 31a.

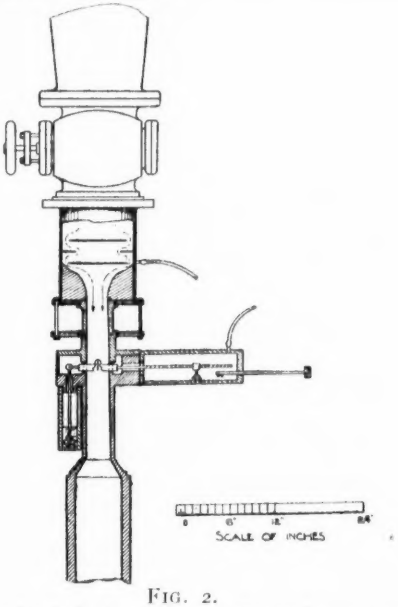

FIG. 2.
High speed wind channel. General arrangement.

The same effects were noticed in the case of the circular arc aerofoil. With this section irrotational flow could be maintained to a higher velocity, the breakdown occurring at a speed greater than 0.7a. This aerofoil showed a definite advantage at high speed over the more conventional R.A.F. 31a. Detailed results are shown in Fig. 3, which are reproduced from the paper already cited.

Since the tunnel itself has a diameter of only 3in., the models have to be constructed on a small scale. In the above experiments the chords of two sets of aerofoils tested were $\frac{1}{2}$in. and $\frac{1}{4}$in. respectively. To construct models on such a small scale necessitates very accurate workmanship, and as the forces exerted at these terrific speeds are very high, appreciable distortion takes place. Sir Thomas Stanton states that at 5° incidence the maximum load on the circular arc aerofoil is of the order of 90 tons sq./in., and that, although the models were made of the finest tool steel, it was possible to fracture them at high angles of incidence.

There is one point to notice. The resistance of a body moving in a fluid of density ρ, kinematic viscosity ν and elasticity E can be expressed in the form

$$R=\rho v^2 l^2 g\left(\frac{vl}{\nu}, E\right)$$

and since $E/\rho=a^2$, R can be written

$$R=\rho v^2 l^2 f\left(\frac{vl}{\nu}, \frac{v}{a}\right).$$

The non-dimensional factor v/a is the quantity which illustrates the effect of compressibility; hence to find the variation of R due to this factor alone it is necessary that the experiments be performed either at the same value of vl/ν, or in that range of velocities, if it exists, for which the resistance is independent of the Reynolds number. For any particular aerofoil the slope of the lift curve is practically independent of the Reynolds number, and so the general conclusions regarding the previous experimental results are unaltered.

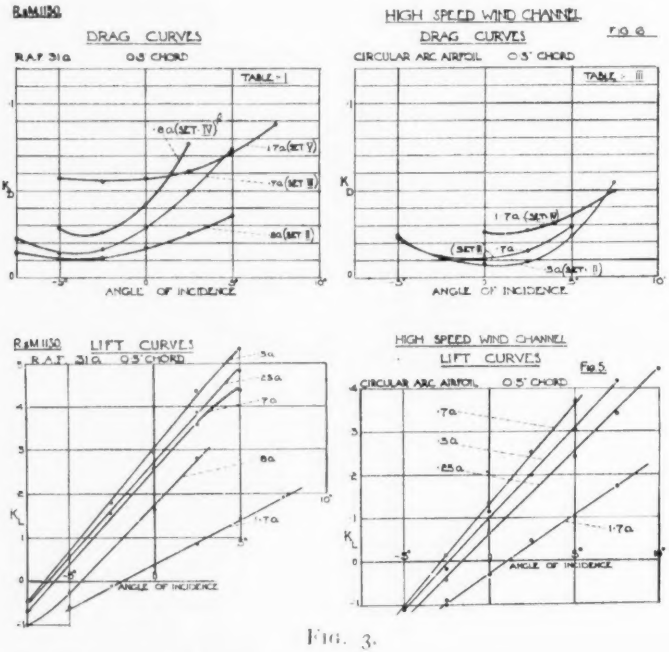

FIG. 3.

To eliminate entirely scale effect, Sir Thomas Stanton performed some experiments upon circular cylinders, and so arranged that as the velocity increased from 0.255a. to 2.20a. the value of vl/ν remained constant. That is to say, he decreased the diameter of the cylinder in the direct proportion that the velocity increased. The results showed that at about 0.4a. the resistance coefficient began to increase and reached about double its normal value, afterwards, when the velocity had increased in 1.5a., falling to the value for normal wind tunnel speeds. The lowest pressure upon the surface of any cylinder appeared to correspond with a velocity not more than 13 per cent. in excess of the local velocity of sound.

It was results such as these which led Prof. G. I. Taylor to investigate in what circumstances the flow of a non-viscous, compressible fluid might remain irrotational. A change from irrotational to rotational motion implies that the motion of an element of fluid changes from one in which it has no angular velocity about its C.G. to one in which the angular velocity has a definite value. The fluid elements then have rotational energy, and the resistance experienced by a body in such circumstances will increase.

The analogy which exists between the flow of an electric current past a hole in a uniform conducting sheet and the motion of a perfect fluid past the

corresponding section is well known. The method has been used for determining the streamlines and the velocity field for two dimensional sections which are not susceptible to mathematical analysis. The lines of current flow are the streamlines, and the velocity at any point is proportional to the electric intensity at that point.

Prof. G. I. Taylor extended this and established the analogy which exists between the flow of an electric current in a thin conducting sheet of variable thickness and the motion of a compressible fluid in which the density is a function of the velocity at the point.

If u and v are the component velocities at a point, the hydrodynamical equations are

$$\left.\begin{aligned} \frac{\partial\phi}{\partial x} &= -u & \frac{\partial\phi}{\partial y} &= -v \\ \frac{\partial\psi}{\partial y} &= -\rho u & \frac{\partial\psi}{\partial x} &= +\rho v \end{aligned}\right\} \quad . \quad . \quad . \quad (6)$$

where ϕ is the velocity potential and the curves ψ=constant are streamlines. The first two equations are the mathematical expression that the vorticity is zero at every point, or that the motion is irrotational; while the last two are derived from the equation of continuity. The equations determining the flow of electric current in a conducting sheet of variable thickness t and constant specific resistance σ are

$$\left.\begin{aligned} \frac{\partial V}{\partial x} &= -f\sigma & \frac{\partial V}{\partial y} &= -g\sigma \\ \frac{\partial W}{\partial x} &= tg & \frac{\partial W}{\partial y} &= -tf \end{aligned}\right\} \quad . \quad . \quad . \quad (7)$$

where f and g are the components of current density. V is the electric potential and W=const. are the lines of current flow.

Comparing the two sets of equations (6) and (7) we find they can be made identical in two ways

(A) If $V=\phi$ and $W=\psi$ in which case

$$t=\rho\sigma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

(B) If $W=\phi$ $V=-\psi$ and

$$t=\sigma/\rho.$$

Analogy (A) is used for representing irrotational motion without circulation, while if a circulation is necessary, as in the case of aerofoils, (B) may be used.

Using analogy (A) the flow of a compressible fluid is identical with the flow of electric current provided the thickness of the conducting sheet is everywhere proportional to the density. Furthermore, in this case the velocity is proportional to the electric force and the lines of current flow are the streamlines in the compressible fluid.

Before the tank can be actually used to solve any particular problem it is necessary to know how the density of the fluid varies with its velocity. The pressure and density in any media are connected by known laws—for example, in the case of air, the isothermal and adiabatic laws.

Bernoulli's integral connects the pressure in the fluid with its velocity and density, and so, by combining this with the appropriate relation between pressure and density, we can determine the relation between density and velocity. At the high speeds involved the natural assumption is that the pressure and density are related adiabatically, and in this case

$$p/p_0=\left(\frac{\rho}{\rho_0}\right)\gamma$$

where p_0 and ρ_0 refer to a standard position and γ is the ratio of the specific heats, which for air has the value

$$\gamma = 1.408$$

Bernoulli's equation is

$$\int \frac{dp}{\rho} + \tfrac{1}{2}q^2 = \text{const.}$$

and by eliminating p between these last two equations we obtain the relation

$$\frac{\rho}{\rho_0} = \left\{ 1 - \tfrac{1}{2}(\gamma - 1)\left(\frac{U^2}{a_0^2}\right)\left(\frac{q^2}{U^2} - 1\right)\right\}^{\frac{1}{\gamma - 1}} \quad . \quad . \quad . \quad (9)$$

p_0, ρ_0 and a_0 are the pressure, density and velocity of sound at infinity, where the fluid is supposed moving with constant velocity U. With these definitions the equation gives the density ρ where the resultant fluid velocity is q.

One interesting result is at once obvious from equation (9). The density ρ will be zero when the velocity is such that

$$1 - \tfrac{1}{2}(\gamma - 1)\left(\frac{U^2}{a_0}\right)\left(\frac{q^2}{U^2} - 1\right) = 0$$

or

$$q^2_{\max} = \left\{ \frac{2}{(\gamma - 1)} \right\} a_0^2 + U^2$$

At this velocity $q_{\max}$ the pressure and density of the air will be zero, and it is, therefore, the limiting speed which the stream of air can attain. Velocities approaching this limit have been obtained in the high speed wind tunnel described earlier.

Let us now consider how the electrical analogy is made use of to determine the flow at high speed past a circular cylinder. The fluid is supposed moving with a uniform velocity U at infinity, and hence at great distances from the cylinder the density ρ_0 will be constant. Using analogy (A) the conducting sheet must be of uniform thickness t_0 at infinity, and as the velocity changes, due to the presence of the cylinder, the sheet must be made thicker or thinner according to the relation $\rho/\rho_0 = t/t_0$. When this has been done the electrical measurements give the magnitude and direction of the velocity at every point.

The apparatus used is illustrated diagrammatically in Fig. 4. The conducting medium consists of a dilute solution of copper sulphate and the current is made alternating to eliminate polarisation effects. The thickness of the sheet is made variable by making the bottom of the tank of paraffin wax, which can then be cut to any desired shape. The cylinder is made of non-conducting material and is placed in the centre of the tank, which consists of a shallow dish rigidly and accurately mounted horizontal. The current is supplied from an alternator to two copper strips, one at each end of the tank and sufficiently far from the centre to be regarded as being at infinity. Two small electrodes, which can be located at any desired point, are used for measuring the direction and magnitude of the electric intensity at the point. The small current which these electrodes take is rectified by a commutator on the same spindle as the current is made to alternate, and is measured by a sensitive galvanometer.

The method of using the tank is now as follows. The conducting sheet is first made of uniform thickness everywhere, and the current adjusted to give the appropriate velocity at infinity. Electrical measurements then determine the velocity at every point in the tank and this corresponds to the perfect fluid flow. The values of q/U found are then substituted in the formula (9) given above, and this gives the density which a compressible fluid moving with this velocity would have. The thickness of the conducting sheet is now changed according to the relation $\rho/\rho_0 = t/t_0$.

This can be done quite easily by adding, or carving away, wax where necessary. The alteration in the thickness of the sheet changes the velocity field, and a new set of electrical measurements are made to give the new velocities. Again, the corresponding density is obtained from (9) and again the sheet is altered. The process can be repeated indefinitely, and if it converges then it necessarily gives the solution of the problem.

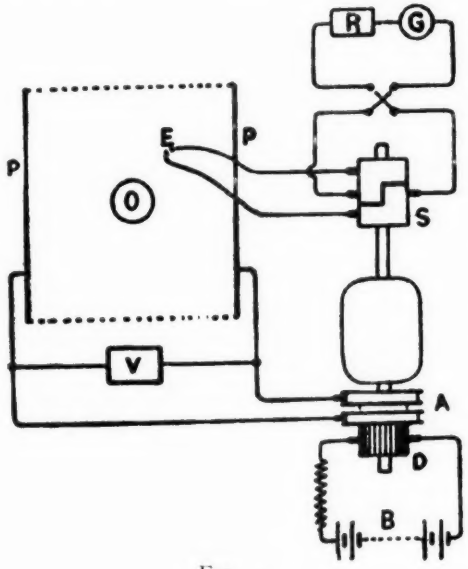

FIG. 4.

In the case of the circular cylinder Prof. Taylor found that his machine gave rapidly convergent results so long as

$$\frac{U}{a_0} < 0.45.$$

That is to say, the successive amounts by which the thickness of the sheet had to be changed became smaller and smaller, and it was possible to carve the bottom to represent the actual distribution of density.

When $U/a_0 > 0.45$ the successive approximations were divergent, and it was no longer possible to carve the bottom of the tank so that the flow of the current represented the flow of the compressible fluid. The conclusion was that either an irrotational solution was impossible, or the tank was incapable of finding it.

Prof. Taylor and Dr. Sharman also tested aerofoil sections, using analogy B. Again they found that for a certain speed at infinity, less than the velocity of sound, the successive approximations diverged. In every case the breakdown appeared to be associated with the attainment by the fluid of the local velocity of sound.

The critical speed at infinity depends in general upon the shape of the body. For the case of the circular cylinder it was approximately $U/a_0 = 0.45$, and for R.A.F. 31a. about $U/a_0 = 0.60$. Referring to the experimental data obtained by Sir Thomas Stanton in the High Speed Tunnel, we noticed that the resistance of a circular cylinder began to increase rapidly as U/a_0 exceeded about 0.40.

Again, with R.A.F. 31a., the lift rapidly decreased and the resistance increased when $U/a_0 > 0.60$. To this extent the results of theory and experiment are in agreement.

The electrical experiments also showed that in the range of velocities for which irrotational motion was possible in the tank the lift coefficient increased as a direct result of the compressibility. This theoretical increase was slightly greater than that actually observed in the tests mentioned earlier.

The practical importance of these results is obvious. Many of the high speed machines of to-day run with propeller tip speeds exceeding the velocity of sound. The experimental and theoretical evidence both point to the fact that these high propeller speeds are undesirable and definitely bad practice. One way to overcome the difficulty is by gearing down the propellers; and this has been done in some cases—notably the S.6.